

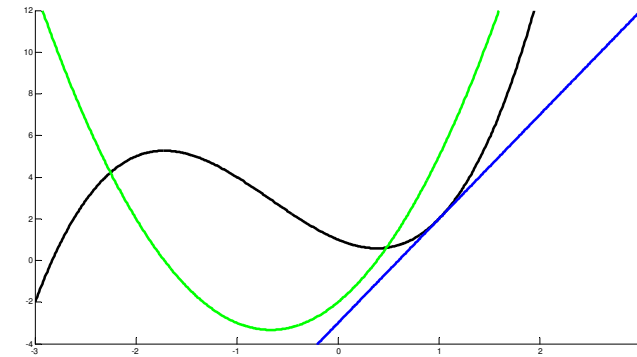
3. prednáška

Backpropagation odvodenie

Priesvitka 1

The Steepest Descent Gradient Method

$$\begin{aligned} f(x) &= x^n & f'(x) &= nx^{(n-1)} \\ f(x) &= e^x & f'(x) &= e^x \\ f(x) &= x^3 + 2x^2 - 2x + 1 & f'(x) &= 3x^2 + 4x - 2 \end{aligned}$$



Priesvitka 2

Chain rule

$$\varphi(x) = f(x) + g(x)$$

$$\varphi'(x) = f'(x) + g'(x)$$

$$\varphi(x) = (f \circ g)(x)$$

$$\varphi'(x) = (f' \circ g)(x)g'(x)$$

$$\varphi(x) = f(g(x))$$

$$\varphi'(x) = f'(g(x))g'(x)$$

$$\varphi(x) = f(g(h(l(x))))$$

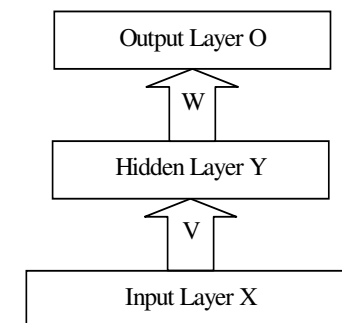
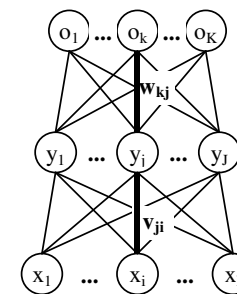
$$\varphi'(x) = f'(g(h(l(x))))g'(h(l(x)))h'(l(x))l'(x)$$

$$f(x) = \frac{1}{1+e^{-x}}$$

$$\begin{aligned} f'(x) &= -(1+e^{-x})^{-2}e^{-x}(-1) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1}{(1+e^{-x})} \frac{e^{-x}}{(1+e^{-x})} = \\ &= \frac{1}{(1+e^{-x})} \frac{1+e^{-x}-1}{(1+e^{-x})} = \frac{1}{(1+e^{-x})} \left(1 - \frac{1}{(1+e^{-x})}\right) = f(x)(1-f(x)) \end{aligned}$$

Priesvitka 3

Feedforward Neural Network – Multilayer Perceptron



- dvojvrstvová dopredná neurónová sieť
- dopredné šírenie signálu
- spätné šírenie chybového signálu

Priesvitka 4

Dopredné šírenie

- vstupný vektor $\bar{x} = (x_1, \dots, x_i)$,
- vypočítaný výstupný vektor $\bar{o} = (o_1, \dots, o_k)$
- pre skrytý neurón j sa vypočíta jeho „net výstup“ \tilde{y}_j ako $\tilde{y}_j = \sum_{i=1}^I v_{ji} x_i$ a vyprodukuje výstup $y_j = f(\tilde{y}_j)$.
- pre výstupný neurón k sa vypočíta jeho „net výstup“ \tilde{o}_k ako $\tilde{o}_k = \sum_{j=1}^J w_{kj} y_j$ a vyprodukuje výstup $o_k = f(\tilde{o}_k)$.
- v_{ji} je váhové prepojenie spájajúce skrytý neurón j so vstupom i
- w_{kj} je váhové prepojenie spájajúce výstupný neurón k so skrytým neurónom j
- f je aktivačná funkcia
- α rýchlosť učenia

Priesvitka 5

Odvodenie spätného šírenia chyby

- tréningová množina – P vzorov $\bar{x}^p = (x_1^p, \dots, x_i^p), \bar{d}^p = (d_1^p, \dots, d_k^p)$
- minimalizácia celkovej chyby: $E = \sum_{p=1}^P E_p$
- čiastkové chyby: $E^p = \frac{1}{2} \sum_{k=1}^K (d_k^p - o_k^p)^2$
- aktivita výstupných neurónov: $o_k^p = f(\tilde{o}_k^p)$
- „net“ aktivita výstupných neurónov: $\tilde{o}_k^p = \sum_{j=1}^J w_{kj} y_j^p$
- aktivita skrytých neurónov: $y_j^p = f(\tilde{y}_j^p)$
- „net“ aktivita skrytých neurónov: $\tilde{y}_j^p = \sum_{i=1}^I v_{ji} x_i^p$
- zmena váh výstupných neuronov: $\Delta w_{kj} = -\alpha \frac{\partial E}{\partial w_{kj}}$
- zmena váh skrytých neuronov: $\Delta v_{ji} = -\alpha \frac{\partial E}{\partial v_{ji}}$

Priesvitka 6

Odvodenie spätného šírenia chyby – výstupné váhy

$$\frac{\partial E}{\partial w_{kj}} = \sum_{p=1}^P \frac{\partial E^p}{\partial w_{kj}}$$

$$\frac{\partial E^p}{\partial w_{kj}} = \frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} \frac{\partial \tilde{o}_k^p}{\partial w_{kj}}$$

$$\delta^{\text{out } p}_k = \frac{\partial E^p}{\partial \tilde{o}_k^p} = -\frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p}$$

$$\frac{\partial E^p}{\partial o_k^p} = \frac{1}{2} 2(d_k^p - o_k^p)(-1) = -(d_k^p - o_k^p)$$

$$\frac{\partial o_k^p}{\partial \tilde{o}_k^p} = f'(\tilde{o}_k^p)$$

$$\frac{\partial \tilde{o}_k^p}{\partial w_{kj}} = y_j^p$$

$$\frac{\partial E^p}{\partial w_{kj}} = -(d_k^p - o_k^p) f'(\tilde{o}_k^p) y_j^p$$

$$\delta^{\text{out } p}_k = -\frac{\partial E^p}{\partial \tilde{o}_k^p} = -\frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} = (d_k^p - o_k^p) f'(\tilde{o}_k^p)$$

Batch update:

$$\Delta w_{kj} = -\alpha \frac{\partial E}{\partial w_{kj}} = \alpha \sum_{p=1}^P (d_k^p - o_k^p) f'(\tilde{o}_k^p) y_j^p$$

$$\Delta w_{kj} = -\alpha \frac{\partial E}{\partial w_{kj}} = \alpha \sum_{p=1}^P \delta^{\text{out } p}_k y_j^p$$

Iterative update:

$$\Delta w_{kj} = -\alpha \frac{\partial E^p}{\partial w_{kj}} = \alpha (d_k^p - o_k^p) f'(\tilde{o}_k^p) y_j^p$$

$$\Delta w_{kj} = -\alpha \frac{\partial E^p}{\partial w_{kj}} = \alpha \delta^{\text{out } p}_k y_j^p$$

Priesvitka 7

Odvodenie spätného šírenia chyby – skryté váhy

$$\frac{\partial E}{\partial v_{ji}} = \sum_{p=1}^P \frac{\partial E^p}{\partial v_{ji}}$$

$$\frac{\partial E^p}{\partial v_{ji}} = \sum_{k=1}^K \left[\frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} \frac{\partial \tilde{o}_k^p}{\partial y_j^p} \frac{\partial y_j^p}{\partial \tilde{y}_j^p} \frac{\partial \tilde{y}_j^p}{\partial v_{ji}} \right] = \left[\sum_{k=1}^K \frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} \frac{\partial \tilde{o}_k^p}{\partial y_j^p} \right] \frac{\partial y_j^p}{\partial \tilde{y}_j^p} \frac{\partial \tilde{y}_j^p}{\partial v_{ji}}$$

$$\delta^{\text{hid } p}_j = -\frac{\partial E^p}{\partial \tilde{y}_j^p} = -\left[\sum_{k=1}^K \frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} \frac{\partial \tilde{o}_k^p}{\partial y_j^p} \right] \frac{\partial y_j^p}{\partial \tilde{y}_j^p}$$

$$\frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} = \frac{\partial E^p}{\partial \tilde{o}_k^p} = -\delta^{\text{out } p}_k$$

$$\frac{\partial \tilde{o}_k^p}{\partial y_j^p} = w_{kj}$$

$$\frac{\partial y_j^p}{\partial \tilde{y}_j^p} = f'(\tilde{y}_j^p)$$

$$\frac{\partial \tilde{y}_j^p}{\partial v_{ji}} = x_i^p$$

$$\frac{\partial E^p}{\partial v_{ji}} = \sum_{k=1}^K \left[\frac{\partial E^p}{\partial \tilde{o}_k^p} w_{kj} \right] f'(\tilde{y}_j^p) x_i^p$$

$$\delta^{\text{hid } p}_j = -\frac{\partial E^p}{\partial \tilde{y}_j^p} = -\left[\sum_{k=1}^K \frac{\partial E^p}{\partial o_k^p} \frac{\partial o_k^p}{\partial \tilde{o}_k^p} \frac{\partial \tilde{o}_k^p}{\partial y_j^p} \right] \frac{\partial y_j^p}{\partial \tilde{y}_j^p} = \left[\sum_{k=1}^K \delta^{\text{out } p}_k w_{kj} \right] f'(\tilde{y}_j^p)$$

Batch update:

$$\Delta v_{ji} = -\alpha \frac{\partial E}{\partial v_{ji}} = -\alpha \sum_{p=1}^P \left[\sum_{k=1}^K \frac{\partial E^p}{\partial \tilde{o}_k^p} w_{kj} \right] f'(\tilde{y}_j^p) x_i^p$$

$$\Delta v_{ji} = -\alpha \frac{\partial E}{\partial v_{ji}} = \alpha \sum_{p=1}^P \delta^{\text{hid } p}_j x_i^p$$

Iterative update:

$$\Delta v_{ji} = -\alpha \frac{\partial E^p}{\partial v_{ji}} = -\alpha \sum_{k=1}^K \left[\frac{\partial E^p}{\partial \tilde{o}_k^p} w_{kj} \right] f'(\tilde{y}_j^p) x_i^p$$

$$\Delta v_{ji} = -\alpha \frac{\partial E^p}{\partial v_{ji}} = \alpha \delta^{\text{hid } p}_j x_i^p$$

Priesvitka 8

Spätne šírenie

- očakávaný = želaný výstupný vektor, $\vec{d} = (d_1 \dots d_K)$
- minimalizujeme chybu $E = \frac{1}{2} \sum_{k=1}^K (d_k - o_k)^2$
- zmeny výstupných váh určíme podľa $\Delta w_{kj} = -\alpha \frac{\partial E}{\partial w_{kj}} = \alpha \delta^{\text{out}_k} y_j$
- chybový signál δ^{out_k} výstupného neurónu k je definovaný ako $\delta^{\text{out}_k} = f'(\tilde{o}_k)(d_k - o_k)$
- zmeny skrytých váh určíme podľa $\Delta v_{ji} = -\alpha \frac{\partial E}{\partial v_{ji}} = \alpha \delta^{\text{hid}_j} x_i$
- chybový signál δ^{hid_j} skrytého neurónu j je definovaný ako $\delta^{\text{hid}_j} = f'(\tilde{y}_j) \sum_{k=1}^K \delta^{\text{out}_k} w_{kj}$
- úprava váhových prepojení $v_{ji} = v_{ji} + \Delta v_{ji}$ a $w_{kj} = w_{kj} + \Delta w_{kj}$

Priesvitka 9

Sigmoidálna aktivačná funkcia

- často používaná aktivačná funkcia $f(x) = \frac{1}{1 + e^{-x}}$
- jej derivácia môže byť jednoducho určená vzťahom $f'(x) = f(x)(1 - f(x))$
- chybové signály môžu byť vyjadrené ako $\delta^{\text{out}_k} = (d_k - o_k) o_k (1 - o_k)$ a $\delta^{\text{hid}_j} = y_j (1 - y_j) \sum_{k=1}^K w_{kj} \delta^{\text{out}_k}$

Momentum

- úprava váh podľa vzťahov $\Delta v_{ji}(t) = \alpha \delta^{\text{hid}_j} x_i + \beta \Delta v_{ji}(t-1)$ a $\Delta w_{kj}(t) = \alpha \delta^{\text{out}_k} y_j + \beta \Delta w_{kj}(t-1)$
- β je momentum

Priesvitka 10