Cvičenia

Cvičenie 9.1. Definujete maticu koeficientov $\boldsymbol{A}$, stĺpcový vektor neznámych $\boldsymbol{x}$ a stĺpcový vektor konštantných členov $\boldsymbol{b}$ pre systémy

(a)

$$x_1 + x_2 = 1$$
$$2x_1 - x_3 = 0$$
$$A = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(b)

$$x_2 = 1$$
$$x_1 = 0$$
$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(c)

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$x_1 + x_2 - x_3 = 1$$
$$x_1 - x_2 - x_3 = -1$$
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

(d)

$$x_1 + x_2 + x_3 - 2x_4 = 1$$
$$A = \begin{pmatrix} 1 & 1 & 1 & -2 \end{pmatrix},\ x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix},\ b = \begin{pmatrix} 1 \end{pmatrix}$$

Cvičenie 9.2. Pomocou Gaussovej eliminačnej metódy riešte systémy lineárnych rovníc

(a)

$$\begin{array}{rrrcr} x & +y & +z & = & 2 \\ 2x & -2y & -z & = & 2 \\ 3x & +y & -2z & = & -2 \end{array}$$

$$A' = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ \boxed{2} & -2 & -1 & 2 \\ \boxed{3} & 1 & -2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -4 & -3 & -2 \\ 0 & \boxed{-2} & -5 & -8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -4 & -3 & -2 \\ 0 & \boxed{4} & 10 & 16 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -4 & -3 & -2 \\ 0 & 0 & 7 & 14 \end{array}\right)$$

$$7z = 14 \Rightarrow z = 2,\ -4y - 3z = -2 \Rightarrow y = -1,\ x + y + z = 2 \Rightarrow x = 1,\ x = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$

(b)

$$\begin{array}{rrrcr} 2x & +2y & +z & = & 4 \\ x & -y & -z & = & 2 \\ 3x & +y & & = & 6 \end{array}$$

$$A' = \left(\begin{array}{ccc|c} 2 & 2 & 1 & 4 \\ \boxed{1} & -1 & -1 & 2 \\ \boxed{3} & 1 & 0 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 2 & 1 & 4 \\ 0 & 4 & 3 & 0 \\ 0 & \boxed{4} & 9 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 2 & 1 & 4 \\ 0 & 4 & 3 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right)$$

$$y = z = 0,\ x = 2,\ x = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

(c)

$$\begin{array}{rrrrcr} 2x_1 & -x_2 & +5x_3 & +3x_4 & = & 5 \\ x_1 & +x_2 & +4x_3 & +3x_4 & = & 7 \\ x_1 & & +3x_3 & +2x_4 & = & 4 \\ & x_2 & +x_3 & +x_4 & = & 3 \end{array}$$

$$A' = \left(\begin{array}{cccc|c} 2 & -1 & 5 & 3 & 5 \\ 1 & 1 & 4 & 3 & 7 \\ 1 & 0 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 4 \\ \boxed{1} & 1 & 4 & 3 & 7 \\ \boxed{2} & -1 & 5 & 3 & 5 \\ 0 & 1 & 1 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 & 3 \\ \cancel{0} & \cancel{-1} & \cancel{-1} & \cancel{-1} & \cancel{-3} \\ \cancel{0} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{3} \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 3 & 2 & 4 \\ 0 & 1 & 1 & 1 & 3 \end{array}\right)$$

$x_4 = l$, $x_3 = k$, $x_2 = 3 - k - l$, $x_1 = 4 - 3k - 2l$, kde $k, l \in \mathbb{R}$,

$$x = \begin{pmatrix} 4 - 3k - 2l \\ 3 - k - l \\ k \\ l \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} -3 \\ -1 \\ 1 \\ 0 \end{pmatrix} + l \begin{pmatrix} -2 \\ -1 \\ 0 \\ 1 \end{pmatrix},$$

(d)

$$\begin{array}{rrcr} 2x & -3y & = & -4 \\ x & +2y & = & 5 \\ -4x & +6y & = & 8 \end{array}$$

$$A' = \left(\begin{array}{rr|r} 2 & -3 & -4 \\ 1 & 2 & 5 \\ -4 & 6 & 8 \end{array}\right) \sim \left(\begin{array}{rr|r} 2 & -3 & -4 \\ 0 & -7 & -14 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{rr|r} 2 & -3 & -4 \\ 0 & 1 & 2 \end{array}\right)$$

$$y = 2 \quad , \ 2x - 3\cdot 2 = -4 \Rightarrow x = 1 , \ x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

(e)

$$\begin{array}{rrrcr} 3x & -2y & +x & = & -4 \\ x & +y & +2z & = & 2 \end{array}$$

$$A' = \left(\begin{array}{rrr|r} 3 & -2 & 1 & -4 \\ 1 & 1 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{rrr|r} 3 & -2 & 1 & -4 \\ \sim 0 & -5 & -5 & -10 \end{array}\right) \sim \left(\begin{array}{rrr|r} 3 & -2 & 1 & -4 \\ \sim 0 & 1 & 1 & 2 \end{array}\right)$$

$$z = k , \ y = 2 - k , \ 3x - 2(2-k) + k = -4 \Rightarrow x = \frac{1}{3}k$$

$$x = \begin{pmatrix} k/3 \\ 2-k \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + k \begin{pmatrix} 1/3 \\ -1 \\ 1 \end{pmatrix}$$

(f)

$$\begin{array}{rrrcr} x_1 & -x_2 & +2x_3 & = & 0 \\ 2x_1 & +3x_2 & -x_3 & = & 0 \end{array}$$

$$A' = \left(\begin{array}{rrr|r} 1 & -1 & 2 & 0 \\ 2 & 3 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{rrr|r} 1 & -1 & 2 & 0 \\ 0 & 5 & -5 & 0 \end{array}\right)$$

$$z = k , \ y = k , \ x = -k$$

$$x = k \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

Cvičenie 9.3. Vypočítajte determinanty matíc:

(a)

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ |\boldsymbol{A}| = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{matrix} = -1$$

(b)

$$\boldsymbol{A}=\begin{pmatrix}1&1&-1\\0&0&0\\1&0&1\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}1&1&-1\\ \cancel{0}&\cancel{0}&\cancel{0}\\1&0&1\end{vmatrix}=0$$

(c)

$$\boldsymbol{A}=\begin{pmatrix}2&1&0&-1\\4&2&5&-1\\3&1/2&-1&2\\0&0&1&1\end{pmatrix}$$

$$|\boldsymbol{A}|=\begin{vmatrix}2&1&0&-1\\4&2&5&-1\\3&1/2&-1&2\\0&0&1&1\end{vmatrix}=-\begin{vmatrix}-1&1&0&2\\-1&2&5&4\\2&1/2&-1&3\\1&0&1&0\end{vmatrix}=\begin{vmatrix}-1&2&0&1\\ \boxed{-1}&4&5&2\\ \boxed{2}&3&-1&1/2\\ \boxed{1}&0&1&0\end{vmatrix}=\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&\boxed{7}&-1&5/2\\0&\boxed{2}&1&1\end{vmatrix}$$

$$=\frac{7}{2}\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&-2&2/7&-5/7\\0&-2&-1&-1\end{vmatrix}=\frac{7}{2}\begin{vmatrix}-1&2&0&1\\0&2&5&1\\0&0&37/7&2/7\\0&0&4&0\end{vmatrix}=-\frac{7}{2}\begin{vmatrix}-1&2&1&0\\0&2&1&5\\0&0&2/7&37/7\\0&0&0&4\end{vmatrix}$$

$$=-\frac{7}{2}(-1)2\frac{2}{7}4=8$$

(d)

$$\boldsymbol{A}=\begin{pmatrix}\sin\alpha&-\cos\alpha\\\cos\alpha&\sin\alpha\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}\sin\alpha&-\cos\alpha\\\cos\alpha&\sin\alpha\end{vmatrix}=\sin^2\alpha+\cos^2\alpha=1$$

(e)

$$\boldsymbol{A}=\begin{pmatrix}1&2&3\\3&1&2\\2&3&1\end{pmatrix},\quad |\boldsymbol{A}|=\begin{vmatrix}1&2&3\\3&1&2\\2&3&1\end{vmatrix}\begin{matrix}1&2\\3&1\\2&3\end{matrix}=(1+8+27)-(6+6+6)=36-18=18$$