Warranted Derivations of Preferred Answer Sets

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What is it all about?

- knowledge representation using rules
- handling preference on rules

Does skippy fly?

- Birds fly.
- Penguins don't fly.
- Penguins are birds.
- Skippy is penguin.

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Introduction: Extended Logic Program

$r_1 \quad fly(skippy) \leftarrow bird(skippy), not\ \neg fly(skippy)$
$r_2 \quad \neg fly(skippy) \leftarrow penguin(skippy), not\ fly(skippy)$
$r_3 \quad bird(skippy) \leftarrow penguin(skippy)$
$r_4 \quad penguin(skippy) \leftarrow$

- default negation: *not a* - there is no evidence that a is true
- explicit negation : $\neg a$ - a is false

$\{penguin(skippy), bird(skippy), fly(skippy)\}$
$\{penguin(skippy), bird(skippy), \neg fly(skippy)\}$

Introduction: Answer set semantics (1)

Basic program (without default negation)

r_1	$\neg fly(skippy)$	$\leftarrow$	$penguin(skippy)$
r_2	$bird(skippy)$	$\leftarrow$	$penguin(skippy)$
r_3	$penguin(skippy)$	$\leftarrow$	

Iterative evaluation

$A_0 = \emptyset$
$A_1 = \{penguin(skippy)\}$
$A_2 = \{penguin(skippy), bird(skippy), \neg fly(skippy)\}$
$A_3 = A_2$

$\{penguin(skippy), bird(skippy), \neg fly(skippy)\}$

Introduction: Answer set semantics (2)

Program with default negation

r_1	$fly(skippy)$	$\leftarrow bird(skippy), not\ \neg fly(skippy)$
r_2	$\neg fly(skippy)$	$\leftarrow penguin(skippy), not\ fly(skippy)$
r_3	$penguin(skippy)$	$\leftarrow$
r_4	$bird(skippy)$	$\leftarrow$

1. guess default literals
2. iterative evaluation
3. check for consistency

Introduction: Answer set semantics (3)

Program

r_1	$fly(skippy)$	$\leftarrow$	$bird(skippy), not\ \neg fly(skippy)$
r_2	$\neg fly(skippy)$	$\leftarrow$	$penguin(skippy), not\ fly(skippy)$
r_3	$penguin(skippy)$	$\leftarrow$	
r_4	$bird(skippy)$	$\leftarrow$	

Semantics

$A_0 = \{not\ fly(skippy)\}$

$A_1 = \{not\ fly(skippy), bird(skippy), penguin(skippy)\}$

$A_2 =$
$\{not\ fly(skippy), bird(skippy), penguin(skippy), \neg fly(skippy)\}$

$A_3 = A_2$

Introduction: Answer set semantics (4)

r_1	$fly(skippy)$	$\leftarrow$	$bird(skippy), not\ \neg fly(skippy)$
r_2	$\neg fly(skippy)$	$\leftarrow$	$penguin(skippy), not\ fly(skippy)$
r_3	$penguin(skippy)$	$\leftarrow$	
r_4	$bird(skippy)$	$\leftarrow$	

Semantics

$\{not\ fly(skippy), not\ \neg fly(skippy)\}$

$\{not\ fly(skippy), not\ \neg fly(skippy), bird(skippy), penguin(skippy)\}$

$\{not\ fly(skippy), not\ \neg fly(skippy), bird(skippy), penguin(skippy), \neg fly(skippy), fly(skippy)\}$

Introduction: Prioritized Logic Program

Prioritized Logic Program $(P, \prec)$

- P is logic program with Answer sets semantics,
- $\prec$ is preference on rules (strict partial order on rules of P),

If $r_1 \prec r_2$ it is said that r_2 is more preferred than r_1.

Task

To transfer preference on rules to preference on answer sets

Introduction: Principles - Brewka and Eiter

Principle III

Let $\mathcal{P} = (P, \prec)$ be prioritized logic program. If P has answer set then $\mathcal{P}$ has preferred answer set.

Goal

To define preferred answer set in a way that satisfies Principle III.

Key idea: Arguments

Program

r_1	b	$\leftarrow a, not\ \neg b$	
r_2	$\neg b$	$\leftarrow not\ b$	
r_3	a	$\leftarrow not\ \neg a$	$r_2 \prec r_1$

Arguments for answer sets

$\{not\ \neg a, not\ b\} \rightarrow \{not\ \neg a, not\ b, a, \neg b\}$
$\{not\ \neg a, not\ \neg b\} \rightarrow \{not\ \neg a, not\ \neg b, a, b\}$

- rule r_2 – *not* b is argument for $\neg b$,
- rule r_1 – *not* $\neg b$ is argument for b if a holds,
- hence *not* b is counterargument against *not* $\neg b$ and vice versa.

Key idea: Argumentation framework

Argumentation framework

- rules $\rightarrow$ basic argumentation structures
- preference $\rightarrow$ attacks on basic argumentation structures
- derivation of complex argumentation structures/attacks
- preferred answer set in terms of attacks

Argumentation structures

Simulate computation of answer sets.

Argumentation framework: Notation

Program

$r_1 \qquad a \qquad \leftarrow \neg b, not\ c$

Rule decomposition

- $head(r_1) = a$
- $body(r_1) = \{\neg b, not\ c\}$
- $(body(r_1))^+ = \{\neg b\}$
- $(body(r_1))^- = \{not\ c\}$

Argumentation framework: Working example

Program

r_1	b	$\leftarrow a, not\ \neg b$	
r_2	$\neg b$	$\leftarrow not\ b$	
r_3	a	$\leftarrow not\ \neg a$	$r_2 \prec r_1$

Argumentation framework: Basic argumentation structures

Basic argumentation structure

r is a rule: $head(r) \leftarrow (body(r))^+, (body(r))^-$
$\mathcal{A} = \langle \{head(r)\} \hookleftarrow (body(r))^-; (body(r))^+ \rangle$ is *basic* argumentation structure.

r_1	b	$\leftarrow a, not\ \neg b$
r_2	$\neg b$	$\leftarrow not\ b$
r_3	a	$\leftarrow not\ \neg a$

Basic argumentation structures

$\mathcal{A}_1$	$\langle \{b\} \hookleftarrow \{not\ \neg b\}; \{a\} \rangle$
$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$

Derived Argumentation Structures (1)

R1: $\mathcal{A}_3 = u(\mathcal{A}_1, \mathcal{A}_2)$ - unfolding

Let be $y_1 \in Obj, r_2 \in P$,
$\mathcal{A}_1 = \langle \{y_1\} \hookleftarrow X_1; Z_1 \rangle$,
$\mathcal{A}_2 = \langle \{head(r_2)\} \hookleftarrow (body(r_2))^-; (body(r_2))^+ \rangle$ argumentation structures,
$head(r_2) \in Z_1$,

Then also
$\mathcal{A}_3 = \langle y_1 \hookleftarrow X_1 \cup (body(r_2))^-; (Z_1 \setminus \{head(r_2)\}) \cup (body(r_2))^+ \rangle$
is an argumentation structure.

Derived Argumentation Structures (2)

Argumentation structures

$\mathcal{A}_1$	$\langle\{b\} \hookleftarrow \{not\ \neg b\}; \{a\}\rangle$
$\mathcal{A}_2$	$\langle\{\neg b\} \hookleftarrow \{not\ b\}\rangle$
$\mathcal{A}_3$	$\langle\{a\} \hookleftarrow \{not\ \neg a\}\rangle$

Unfolding

$$\mathcal{A}_4 = u(\mathcal{A}_1, \mathcal{A}_3) \quad = \langle\{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\}\rangle$$

Derived Argumentation Structures (3)

R2: $\mathcal{A}_3 = \mathcal{A}_1 \cup \mathcal{A}_2$ - joining

Let $\mathcal{A}_1 = \langle Y_1 \hookleftarrow X_1 \rangle$ and $\mathcal{A}_2 = \langle Y_2 \hookleftarrow X_2 \rangle$ be argumentation structures.
Then also $\mathcal{A}_3 = \langle Y_1 \cup Y_2 \hookleftarrow X_1 \cup X_2 \rangle$ is an argumentation structure.

$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$
$\mathcal{A}_4$	$\langle \{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$

$$\mathcal{A}_5 = \mathcal{A}_3 \cup \mathcal{A}_2 = \langle \{a, \neg b\} \hookleftarrow \{not\ \neg a, not\ b\} \rangle$$
$$\mathcal{A}_6 = \mathcal{A}_3 \cup \mathcal{A}_4 = \langle \{a, b\} \hookleftarrow \{not\ \neg a, not\ \neg b\} \rangle$$

Derived Argumentation Structures (4)

R3: $\mathcal{A}_2 = \mathcal{A}_1 \cup W$ - extending

Let $\mathcal{A}_1 = \langle Y_1 \hookleftarrow X_1 \rangle$ be an argumentation structure.
Then also $\mathcal{A}_2 = \langle Y_1 \hookleftarrow X_1 \cup W \rangle$ is an argumentation structure.

$$\begin{aligned} \mathcal{A}_3 &= \langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle \\ W &= \{not\ b\} \end{aligned}$$

$$\mathcal{A}_7 = \mathcal{A}_3 \cup W = \langle \{a\} \hookleftarrow \{not\ \neg a, not\ b\} \rangle$$

Example: Derived argumentation structures

Basic argumentation structures

$\mathcal{A}_1$	$\langle \{b\} \hookleftarrow \{not\ \neg b\}; \{a\} \rangle$
$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$

Derived argumentation structures

$\mathcal{A}_4 = u(\mathcal{A}_1, \mathcal{A}_3)$	$\langle \{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$
$\mathcal{A}_5 = \mathcal{A}_3 \cup \mathcal{A}_2$	$\langle \{a, \neg b\} \hookleftarrow \{not\ b, not\ \neg a\} \rangle$
$\mathcal{A}_6 = \mathcal{A}_3 \cup \mathcal{A}_4$	$\langle \{a, b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$

Answer sets

$\{a, \neg b, not\ b, not\ \neg a\}$

$\{a, b, not\ \neg b, not\ \neg a\}$

Argumentation framework: Complete arguments

Complete arguments

An argumentation structure $\langle Y \hookleftarrow X \rangle$ is called *complete* iff for each literal $L \in Obj$ it holds that $L \in Y$ or $not\ L \in X$.

Complete argumentation structures correspond to Answer sets.

Argumentation framework: Basic attacks

Basic argumentation structures

$\mathcal{A}_1$	$\langle \{b\} \hookleftarrow \{not\ \neg b\}; \{a\} \rangle$
$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$

- A_1 contradicts A_2,
- A_2 contradicts A_1.
- $r_2 \prec r_1$ for corresponding rules

Basic attacks

A_1 attacks A_2

Argumentation framework: Derived Attacks (1)

Q2 - left side is unfolded

Let $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ be argumentation structures such that:

- $\mathcal{A}_1$ attacks $\mathcal{A}_2$,
- $\mathcal{A}_3$ is not attacked, and
- $u(\mathcal{A}_1, \mathcal{A}_3)$ is argumentation structure.

Then $u(\mathcal{A}_1, \mathcal{A}_3)$ attacks $\mathcal{A}_2$.

$$
\begin{array}{ll}
\mathcal{A}_1 & \langle \{b\} \hookleftarrow \{not\ \neg b\}; \{a\} \rangle \\
\mathcal{A}_2 & \langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle \\
\mathcal{A}_3 & \langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle \\
\mathcal{A}_4 = u(\mathcal{A}_1, \mathcal{A}_3) & \langle \{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle
\end{array}
$$

$\mathcal{A}_1$ attacks $\mathcal{A}_2$, $\mathcal{A}_4$ attacks $\mathcal{A}_2$

Argumentation framework: Derived Attacks (2)

Q3 - left side is joined

Let $\mathcal{A}_1$ and $\mathcal{A}_3$ be argumentation structures of the form $\langle X \hookleftarrow Y \rangle$ and $\mathcal{A}_2$ be an argumentation structure. Suppose that:

- $\mathcal{A}_1$ attacks $\mathcal{A}_2$,
- $\mathcal{A}_3$ is not attacked and
- $\mathcal{A}_1 \cup \mathcal{A}_3$ is argumentation structure.

Then $\mathcal{A}_1 \cup \mathcal{A}_3$ attacks $\mathcal{A}_2$.

$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$
$\mathcal{A}_4$	$\langle \{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$
$\mathcal{A}_6 = \mathcal{A}_3 \cup \mathcal{A}_4$	$\langle \{a, b\} \hookleftarrow \{not\ \neg a, not\ \neg b\} \rangle$

$\mathcal{A}_4$ attacks $\mathcal{A}_2$, $\mathcal{A}_6$ attacks $\mathcal{A}_2$

Argumentation framework: Derived Attacks (3)

Q4 - right side is joined

Let $\mathcal{A}_1$ be an argumentation structure and $\mathcal{A}_2$, $\mathcal{A}_3$ be argumentation structures of the form $\langle X \hookleftarrow Y \rangle$ such that:

- $\mathcal{A}_1$ attacks $\mathcal{A}_2$,
- $\mathcal{A}_3$ does not attack $\mathcal{A}_1$, and
- $\mathcal{A}_2 \cup \mathcal{A}_3$ is argumentation structure.

Then $\mathcal{A}_1$ attacks $\mathcal{A}_2 \cup \mathcal{A}_3$.

$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$
$\mathcal{A}_5 = \mathcal{A}_3 \cup \mathcal{A}_2$	$\langle \{a, \neg b\} \hookleftarrow \{not\ b, not\ \neg a\} \rangle$

$\mathcal{A}_6$ attacks $\mathcal{A}_2$, $\mathcal{A}_6$ attacks $\mathcal{A}_5$

Example: Derived attacks

Argumentation structures

$\mathcal{A}_1$	$\langle \{b\} \hookleftarrow \{not\ \neg b\}; \{a\} \rangle$
$\mathcal{A}_2$	$\langle \{\neg b\} \hookleftarrow \{not\ b\} \rangle$
$\mathcal{A}_3$	$\langle \{a\} \hookleftarrow \{not\ \neg a\} \rangle$
$\mathcal{A}_4 = u(\mathcal{A}_1, \mathcal{A}_3)$	$\langle \{b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$
$\mathcal{A}_5 = \mathcal{A}_3 \cup \mathcal{A}_2$	$\langle \{a, \neg b\} \hookleftarrow \{not\ b, not\ \neg a\} \rangle$
$\mathcal{A}_6 = \mathcal{A}_3 \cup \mathcal{A}_4$	$\langle \{a, b\} \hookleftarrow \{not\ \neg b, not\ \neg a\} \rangle$

Basic attacks

A_1 attacks A_2

Derived attacks

A_4 attacks A_2, A_6 attacks A_2, A_6 attacks A_5

Argumentation framework: Preferred answer set

Complete argumentation structures

$\mathcal{A}_5$ $\langle\{a, \neg b\} \hookleftarrow \{not\ b, not\ \neg a\}\rangle$
$\mathcal{A}_6$ $\langle\{a, b\} \hookleftarrow \{not\ \neg b, not\ \neg a\}\rangle$

Attacks on complete argumentation structures

A_6 attacks A_5

Preferred answer sets

$\{a, b, not\ \neg b, not\ \neg a\}$ - A_6 in not attacked by complete argumentation structure

Argumentation framework: Recapitulation

Argumentation framework

- rules $\rightarrow$ basic argumentation structures
- preference $\rightarrow$ attacks on basic argumentation structures
- derivation of complex argumentation structures/attacks
- preferred answer set in terms of attacks

Argumentation framework: Principle satisfaction

Theorem

Principle III is satisfied

Formulation of conditions in attack derivation rules is crucial, e.g

Q2 - left side is unfolded

Let $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ be argumentation structures such that:

- $\mathcal{A}_1$ attacks $\mathcal{A}_2$,
- $\mathcal{A}_3$ is not attacked, and
- $u(\mathcal{A}_1, \mathcal{A}_3)$ is argumentation structure.

Then $u(\mathcal{A}_1, \mathcal{A}_3)$ attacks $\mathcal{A}_2$.

The end