



L-systémy a modelovanie rastlín

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Obsah prezentácie

- Základy L-systémov
- OL-systémy
- IL-systémy
- Parciálne L-systémy
- *Veľa obrázkov*

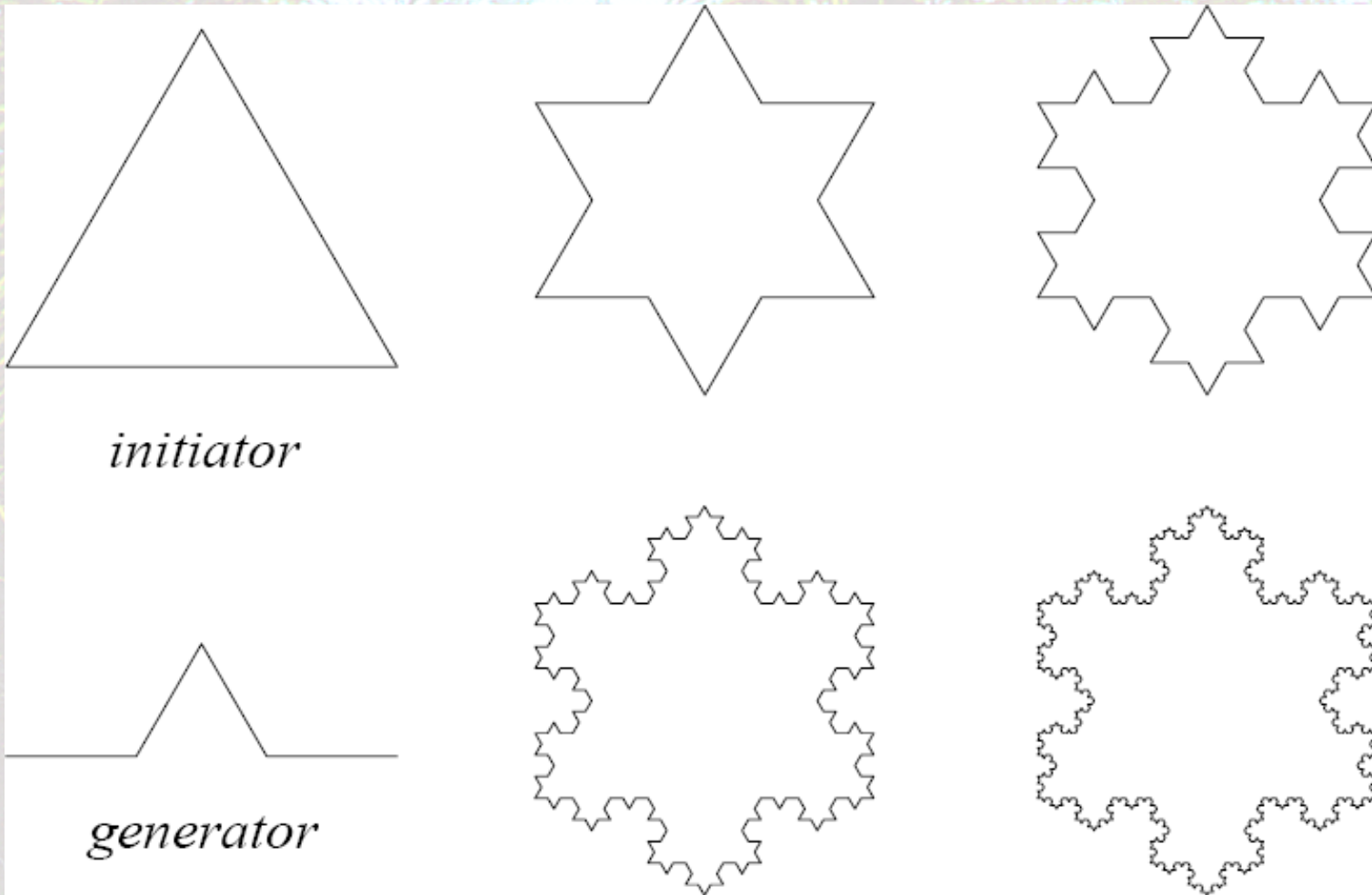
L-systémy

- Inšpirované rastlinami
 - krása symetrie
 - elegancia a jednoduchosť rastu
 - samopodobnosť
- L-systémy
 - matematický formalizmus (1968)
 - pôvodne určené na opis topológie rastlín
 - grafické interpretácie vznikli až neskôr
 - spájajú matematiku, geometriu, biológiu a umenie

L-systémy (2)

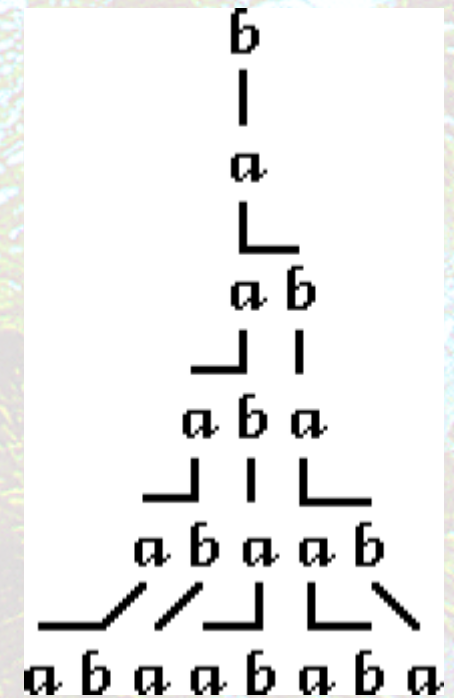
- Základy L-systémov
 - prepisovacie alebo produkčné pravidlá
 - transformácia jednoduchých štruktúr na zložitejšie
 - iniciátor a generátor
- Kategorizácia L-systémov
 - bezkontextové/kontextové (OL/IL)
 - deterministické/stochastické
 - parametrické

Iniciátor a generátor



Deterministické OL systémy

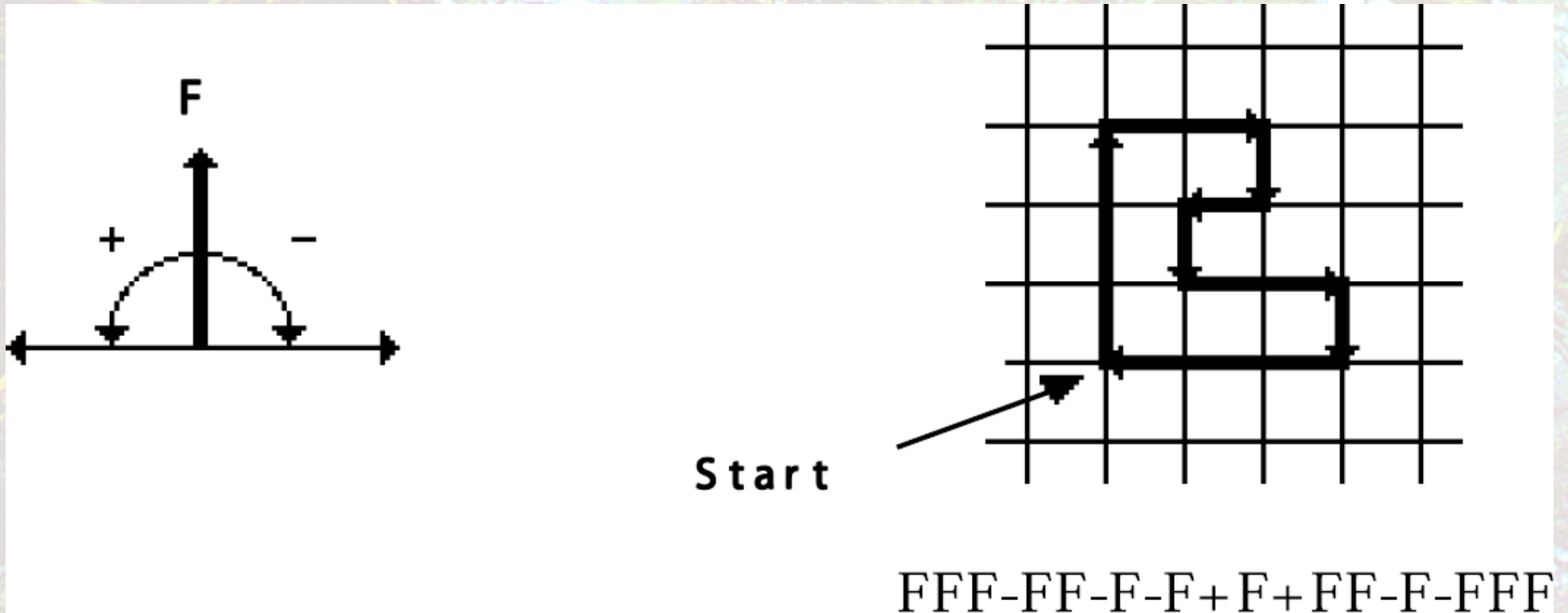
- Iniciátor
 - $w : a$
- Produkčné pravidlá
 - $p1: a \rightarrow ab$
 - $p2: b \rightarrow a$



Korytnačia grafika

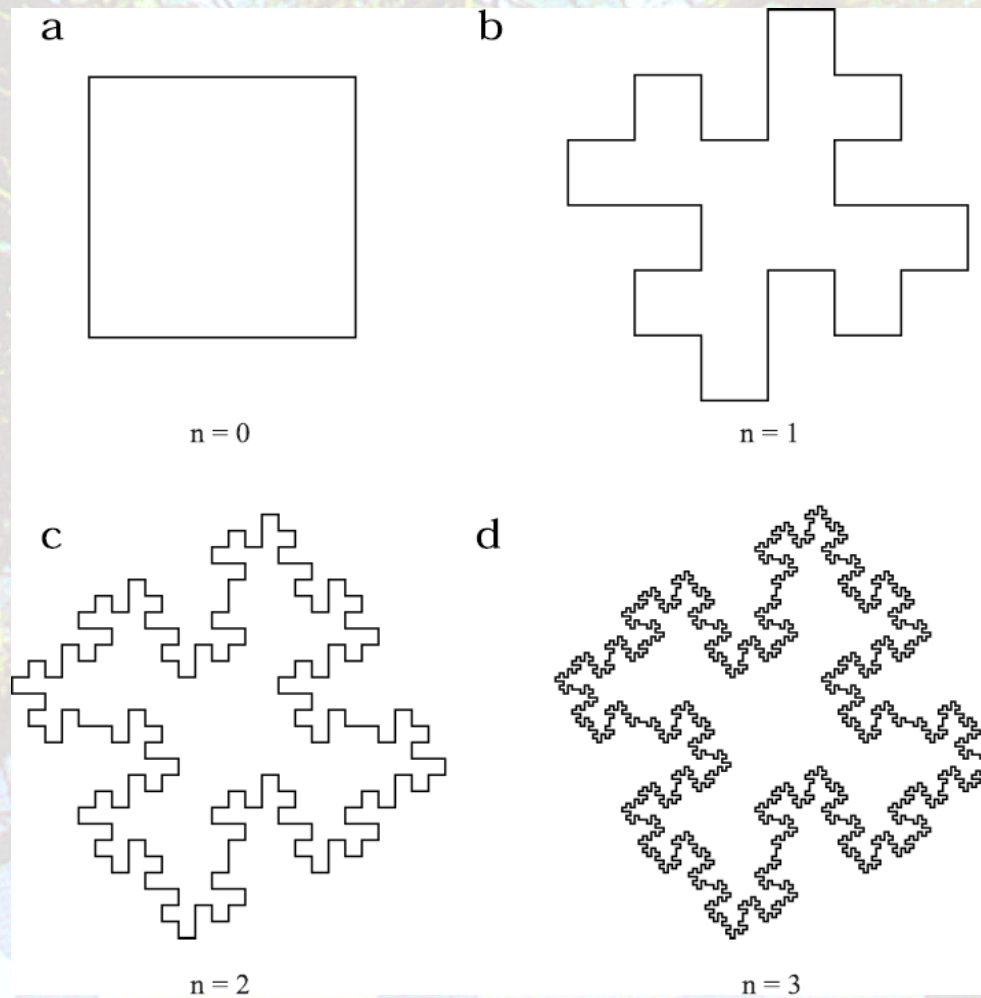
- “Korytnačka” má stav daný
 - pozíciou
 - smerom
- Operácie
 - Posun dopredu
 - s kreslením čiary - symbol “F”
 - bez kreslenia čiary - symbol “f”
 - Rotácia
 - Doľava - symbol “+”
 - Doprava - symbol “-”

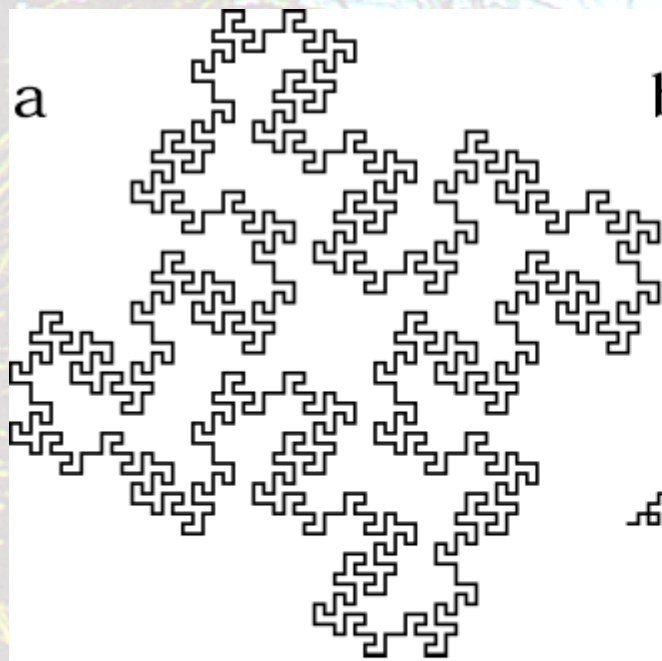
Korytnačia grafika (ukážka)



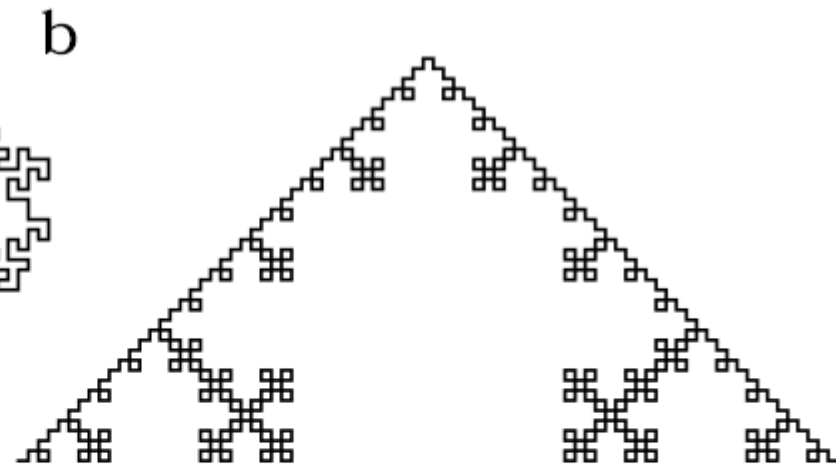
Kvadratický Kochov ostrov

- w: F-F-F-F
- p: F $\rightarrow$ F-F+F+FF-F-F+F

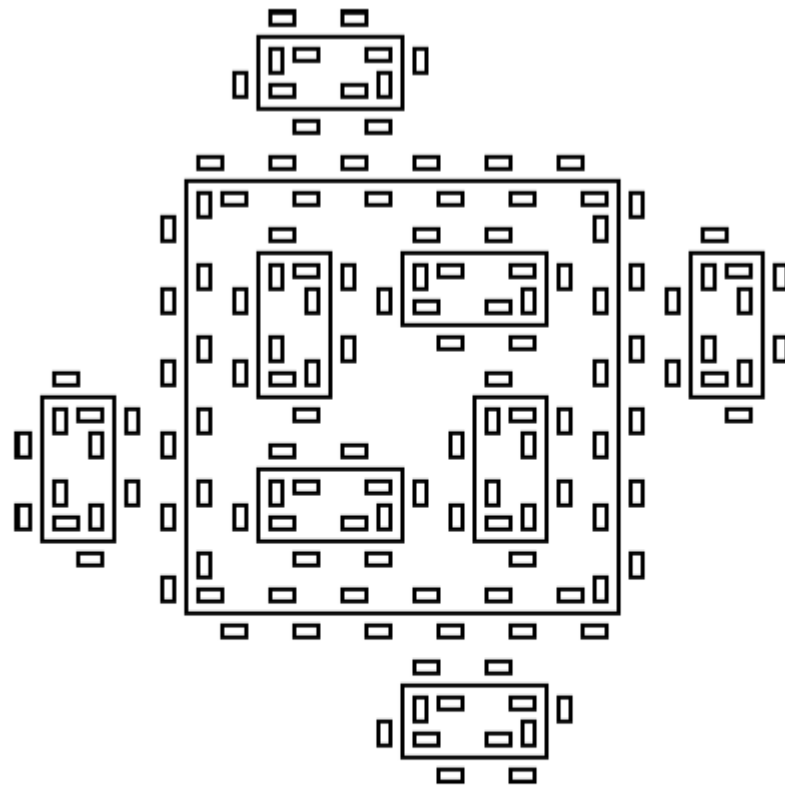




$n = 2, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow F+FF-FF-F-F+F+F$
 $F-F-F+F+FF+FF-F$



$n = 4, \delta = 90^\circ$
 $-F$
 $F \rightarrow F+F-F-F+F$



$$n = 2, \delta = 90^\circ$$

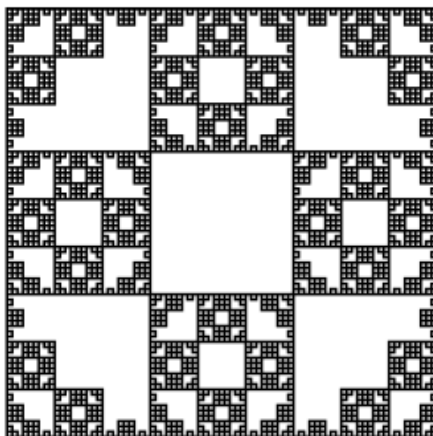
$$F + F + F + F$$

$$F \rightarrow F + f - FF + F + FF + Ff + FF - f + FF - F - FF - Ff - FFF$$

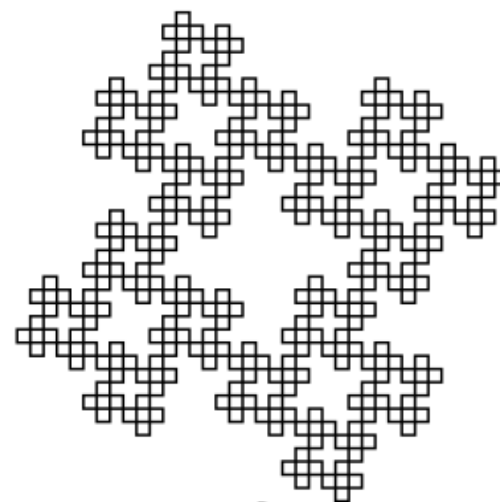
$$f \rightarrow ffffff$$



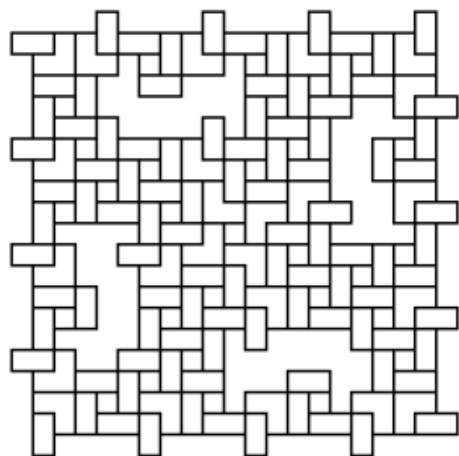
$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F-F-F-F+F$



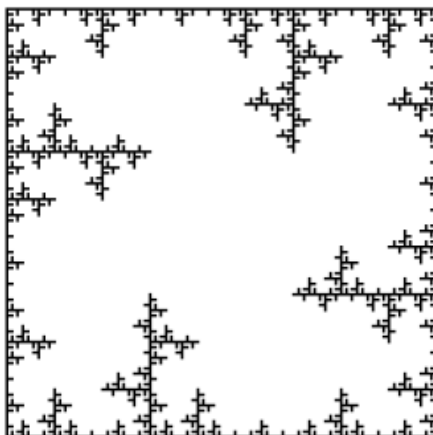
$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F-F-F-FF$



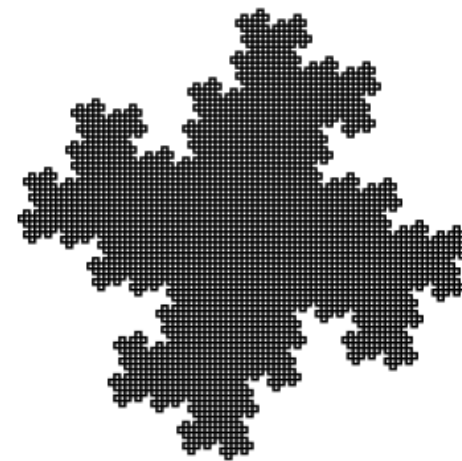
$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow F-F+F-F-F$



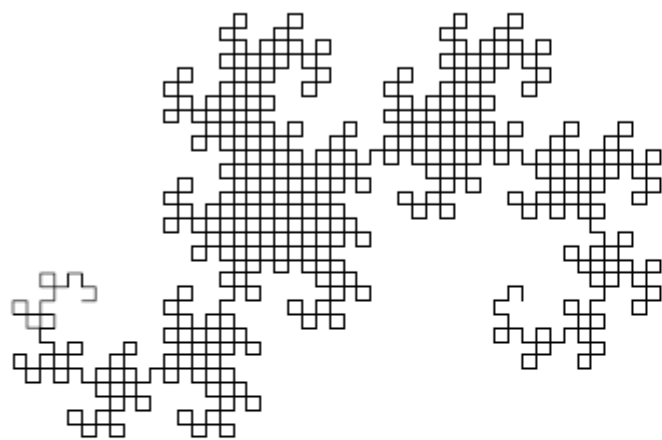
$n = 3, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F+F-F-FF$



$n = 4, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F--F-F$



$n = 5, \delta = 90^\circ$
 $F-F-F-F$
 $F \rightarrow F-FF--F-F$

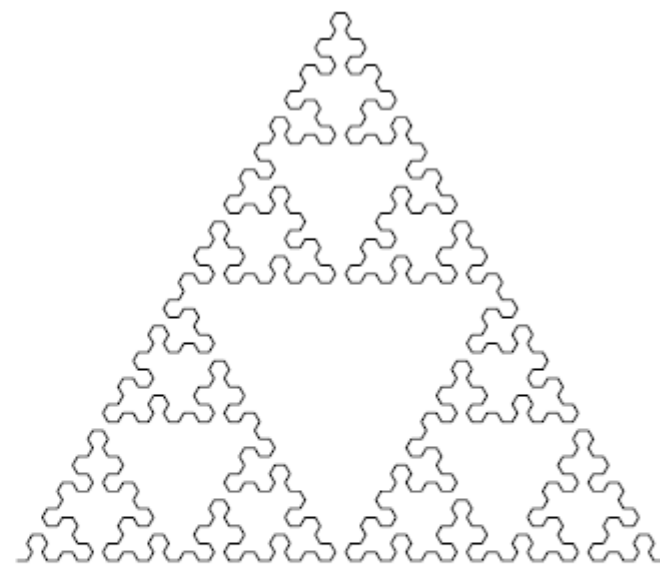


$n=10, \delta=90^\circ$

F_l

$F_l \rightarrow F_l + F_r +$

$F_r \rightarrow -F_l - F_r$



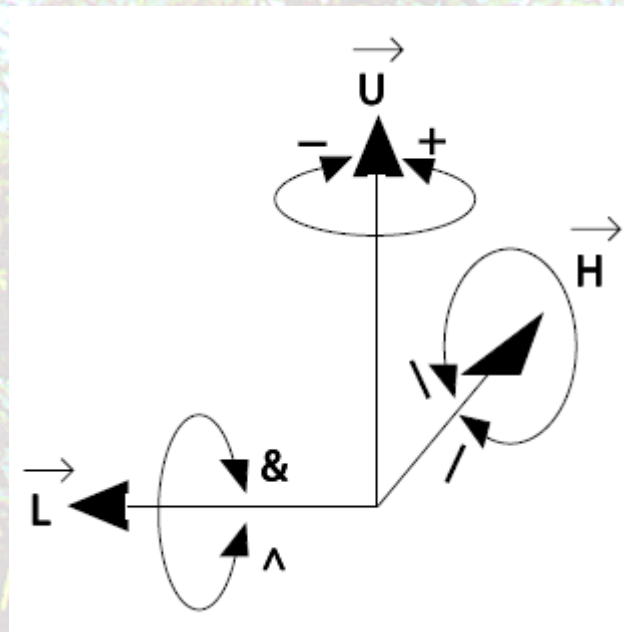
$n=6, \delta=60^\circ$

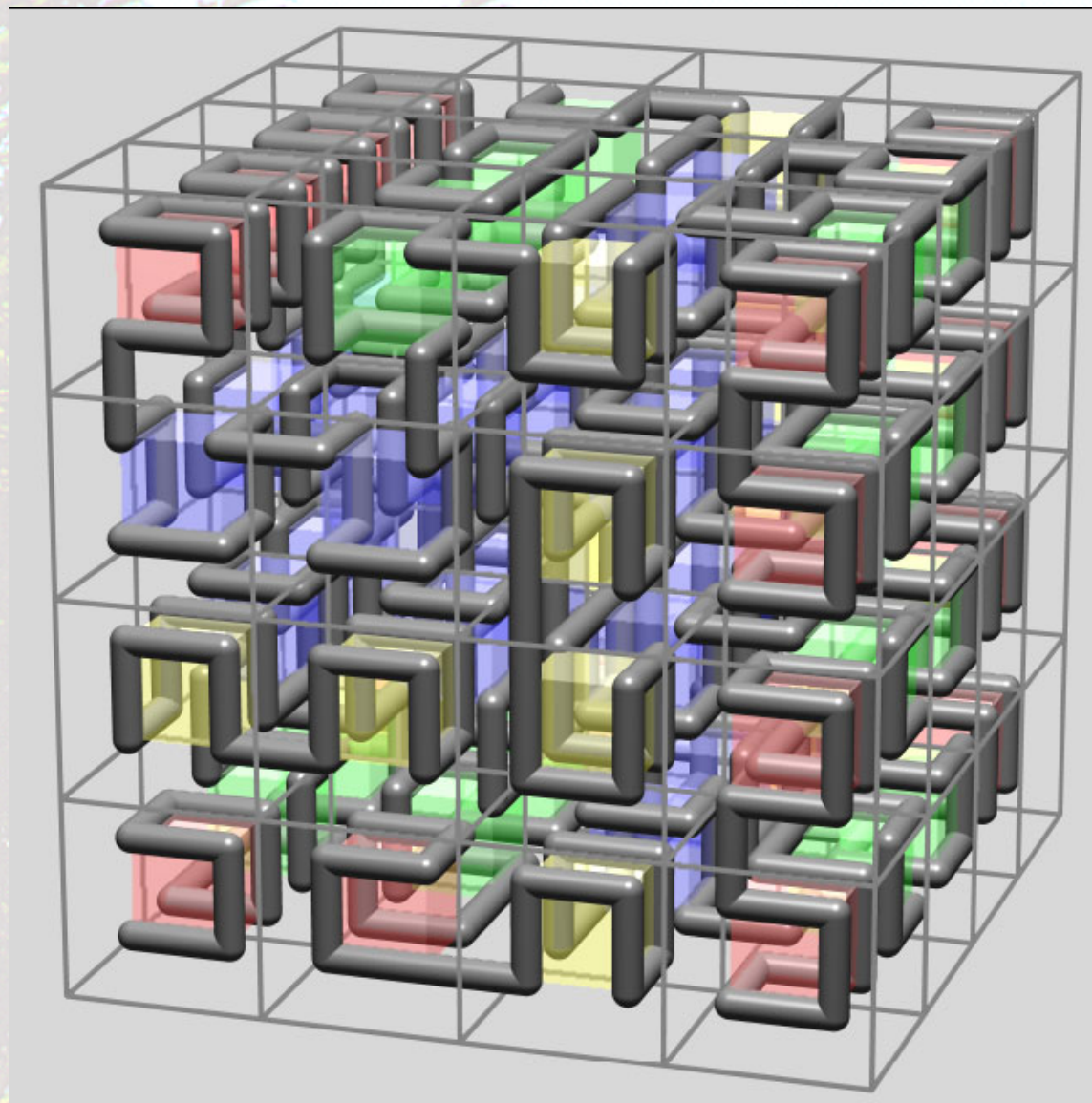
F_r

$F_l \rightarrow F_r + F_l + F_r$

$F_r \rightarrow F_l - F_r - F_l$

Korytnačia grafika 3D





$n=2, \delta=90^\circ$

A

$A \rightarrow B-F+CFC+F-D\&F\wedge D-F+\&\&CFC+F+B//$

$B \rightarrow A\&F\wedge CFB\wedge F\wedge D\wedge\wedge-F-D\wedge|F\wedge B|FC\wedge F\wedge A//$

$C \rightarrow |D\wedge|F\wedge B-F+C\wedge F\wedge A\&\&FA\&F\wedge C+F+B\wedge F\wedge D//$

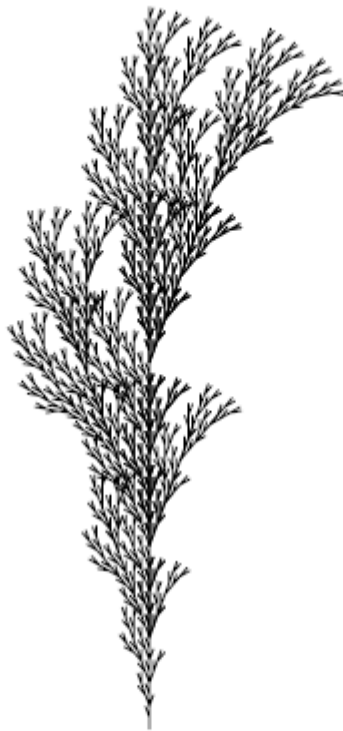
$D \rightarrow |CFB-F+B|FA\&F\wedge A\&\&FB-F+B|FC//$

Zátvorkové L-systémy

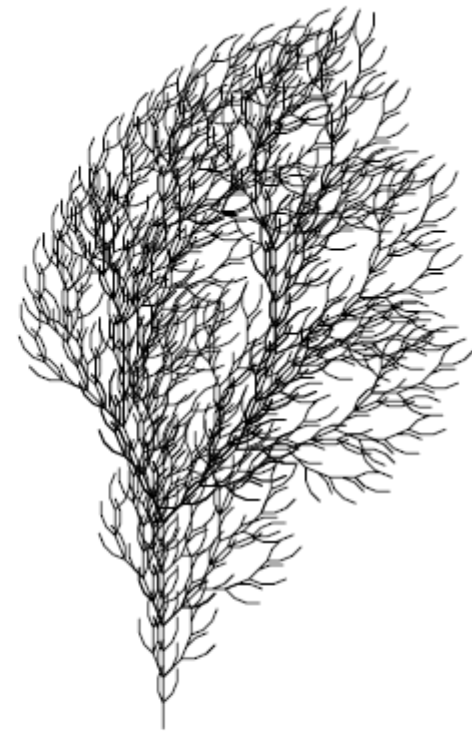
- Dokážu popisovať stromovú topológiu
- Rozšírenie abecedy o špeciálne symboly
 - symbol “[“ - uloženie stavu do zásobníka
 - symbol “]” - získanie stavu zo zásobníka
- *Stav je pozícia, smer,...*



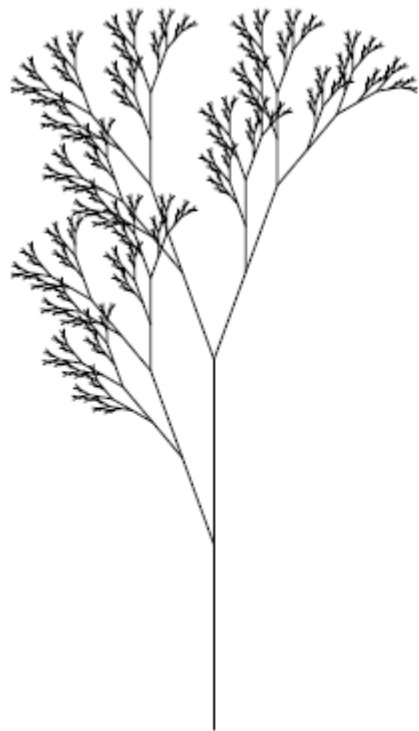
a
 $n=5, \delta=25.7^\circ$
 F
 $F \rightarrow F[+F]F[-F]F$



b
 $n=5, \delta=20^\circ$
 F
 $F \rightarrow F[+F]F[-F][F]$



c
 $n=4, \delta=22.5^\circ$
 F
 $F \rightarrow FF[-F+F+F] +$
 $[+F-F-F]$



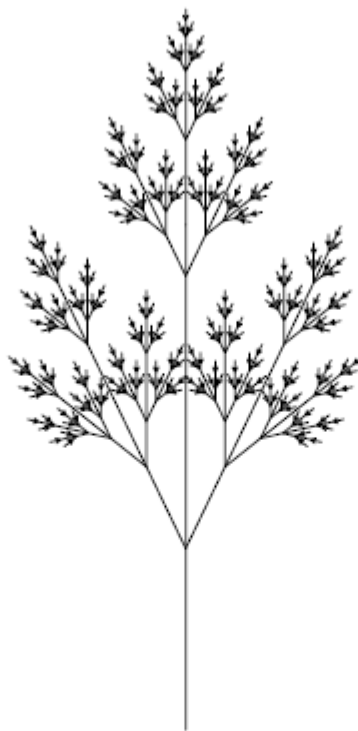
d

$n=7, \delta=20^\circ$

X

$X \rightarrow F[+X]F[-X]+X$

$F \rightarrow FF$



e

$n=7, \delta=25.7^\circ$

X

$X \rightarrow F[+X][-X]FX$

$F \rightarrow FF$



f

$n=5, \delta=22.5^\circ$

X

$X \rightarrow F-[[X]+X]+F[+FX]-X$

$F \rightarrow FF$



$n=7$, $\delta=22.5^\circ$

ω : A
 p_1 : $A \rightarrow [\&FL!A]////////' [\&FL!A]////////' [\&FL!A]$
 p_2 : $F \rightarrow S ///// F$
 p_3 : $S \rightarrow F L$
 p_4 : $L \rightarrow [',',', \wedge \wedge \{-f+f+f-|-f+f+f\}]$

$$n=5, \delta=18^\circ$$
 ω : plant
$$p_1 : \text{plant} \rightarrow \text{internode} + [\text{plant} + \text{flower}] - - //$$

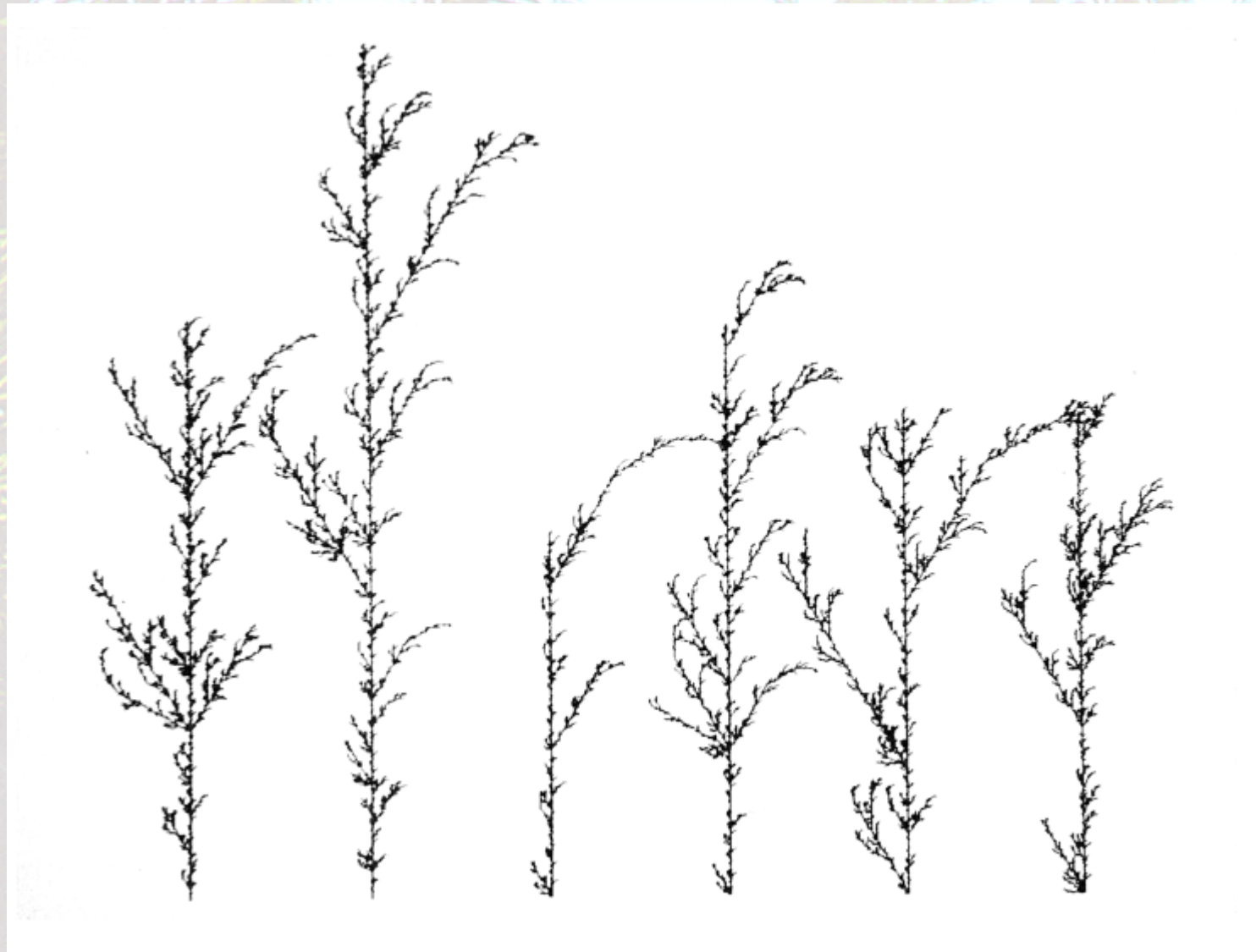
$$[\text{-- leaf}] \text{internode} [\text{++ leaf}] -$$

$$[\text{plant flower}] + + \text{plant flower}$$
$$p_2 : \text{internode} \rightarrow \overline{\text{F}} \text{ seg } [// \ \& \ \& \ \text{leaf}] \ [// \wedge \wedge \ \text{leaf}] \ \text{F seg}$$
$$p_3 : \text{seg} \rightarrow \text{seg F seg}$$
$$p_4: \text{leaf} \rightarrow [^{\text{g}} \{ +\text{f}-\text{ff}-\text{f}+ \mid +\text{f}-\text{ff}-\text{f} \}]$$
$$p_5: \text{flower} \rightarrow \left[\begin{array}{c} \& \& \& \text{pedicel} \text{ ' / wedge } \text{//// wedge } \text{////} \\ \text{wedge } \text{//// wedge } \text{//// wedge } \end{array} \right]$$
$$p_6 : \text{pedicel} \rightarrow \text{FF}$$
$$p_7: \text{wedge} \rightarrow [\wedge F] [\{ \& \& \& \& -f+f \mid -f+f \}]$$

Stochastické L-systémy

- Existuje viac aplikovateľných produkčných pravidiel pre jeden symbol
- Pre “nejednoznačné” produkčné pravidlá sú definované pravdepodobnosti

$$\begin{aligned}\omega &: F \\ p_1 &: F \xrightarrow{.33} F[+F]F[-F]F \\ p_2 &: F \xrightarrow{.33} F[+F]F \\ p_3 &: F \xrightarrow{.34} F[-F]F\end{aligned}$$





Kontextové L-systémy

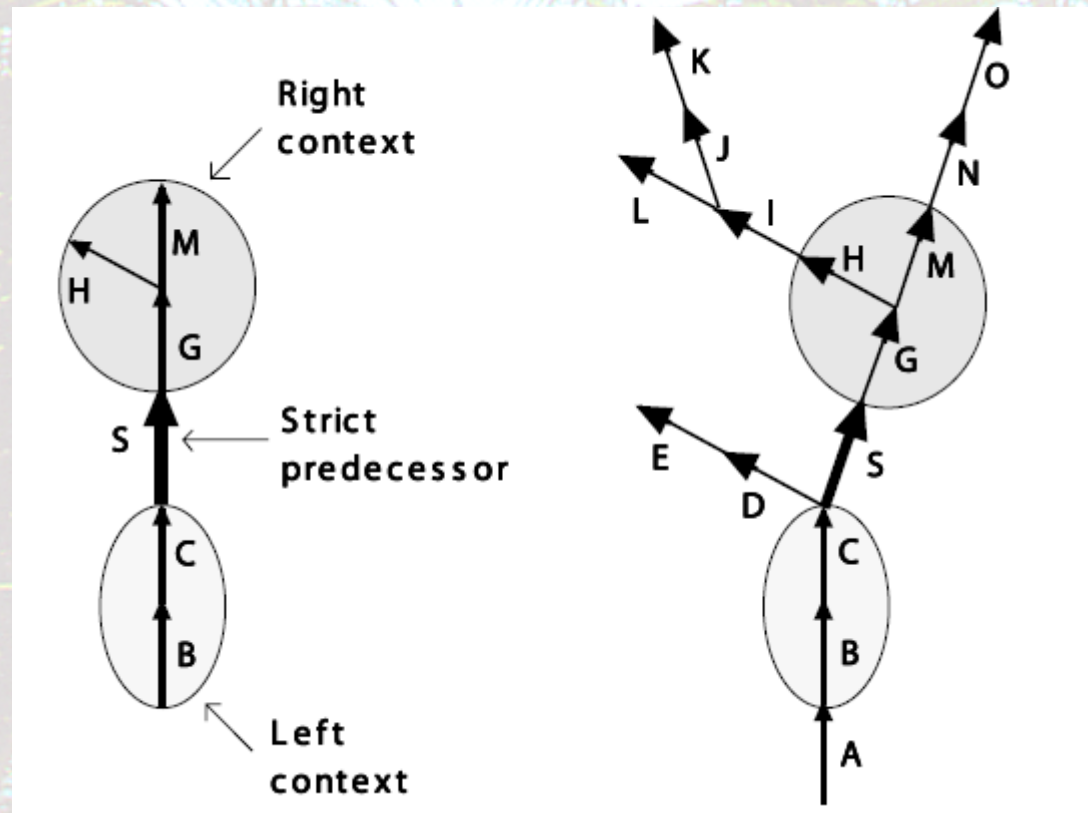
- Produkčné pravidlá citlivé na kontext
- 2L-systémy
 - $p1: A < S > B \rightarrow C$
 - ak pred “S” je “A” a za “S” je “B” potom “S” prepíš na “C”
- 1L-systémy
 - iba jednostranný kontext
- Všeobecne ide o IL alebo (k, l) systémy
 - k je dĺžka ľavého a l je dĺžka pravého kontextu

Ukážka 1L

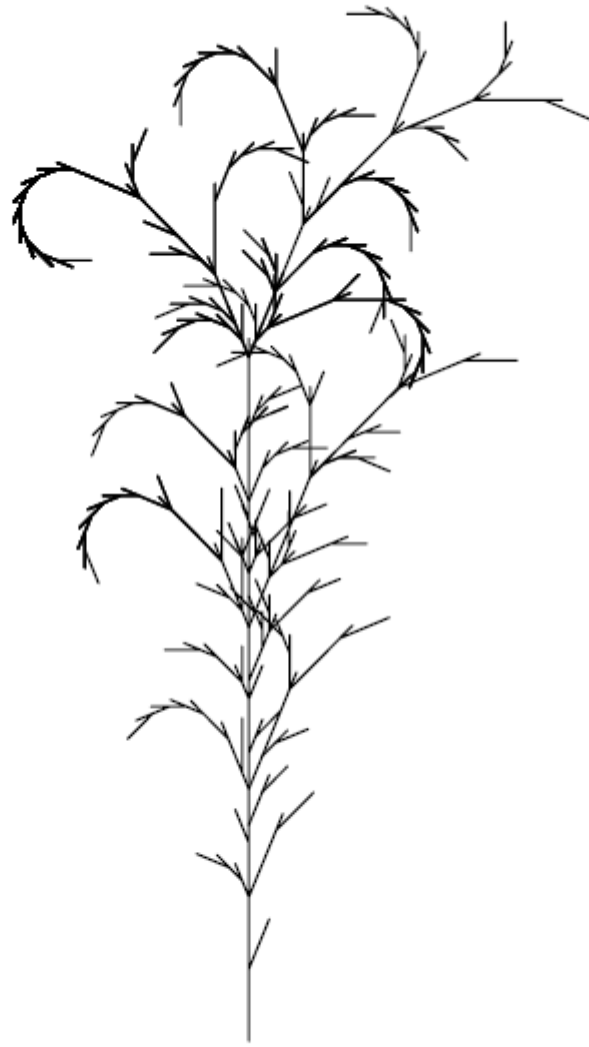
- w: baaaaaaaaa
- p1: $b < a \rightarrow b$
- p2: $b \rightarrow a$
- baaaaaaaaa
- abaaaaaaaaa
- aabaaaaaaaaa
- aaabaaaaaaaaa
- aaaabaaaaaa
- ...

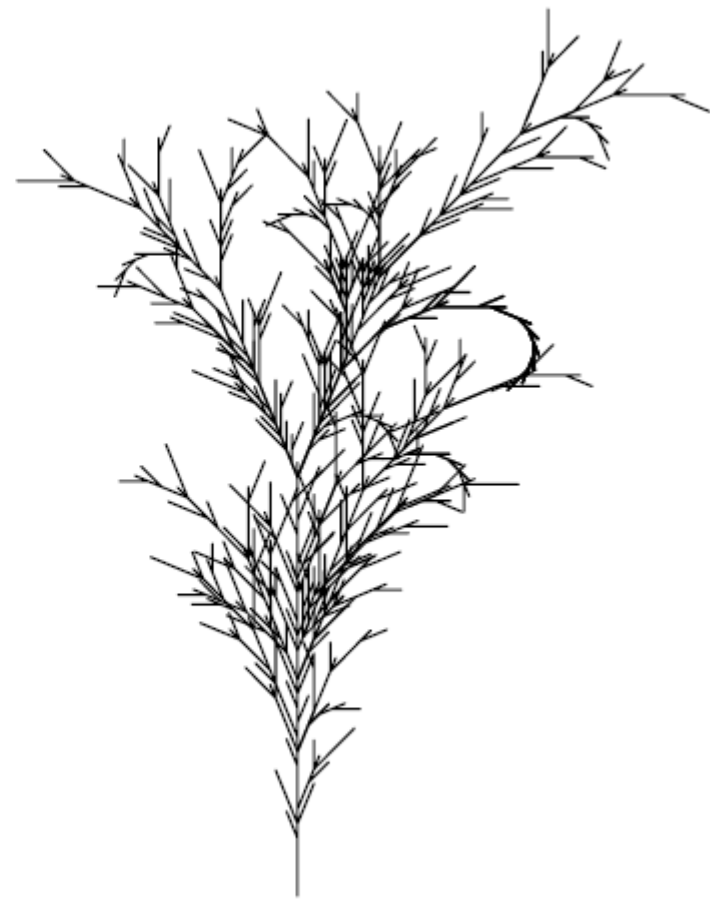
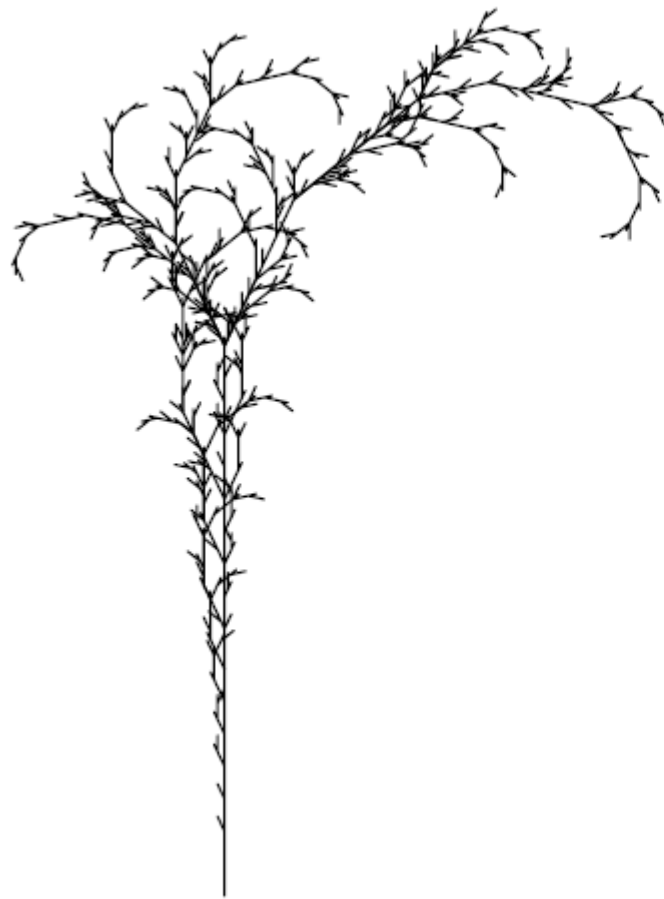
Kontextovo závislé produkčné pravidlá majú prednosť.

Kontext v zátvorkovom systéme



- **ABC[DE][SG[HI[JK]L]MNO]**
- **BC < S > G[H]M**





$n=30, \delta=22.5^\circ$

#ignore: +-F

F1F1F1

0 < 0 > 0 → 0
0 < 0 > 1 → 1[+F1F1]
0 < 1 > 0 → 1
0 < 1 > 1 → 1
1 < 0 > 0 → 0
1 < 0 > 1 → 1F1
1 < 1 > 0 → 0
1 < 1 > 1 → 0
* < + > * → -
* < - > * → +

$n=30, \delta=22.5^\circ$

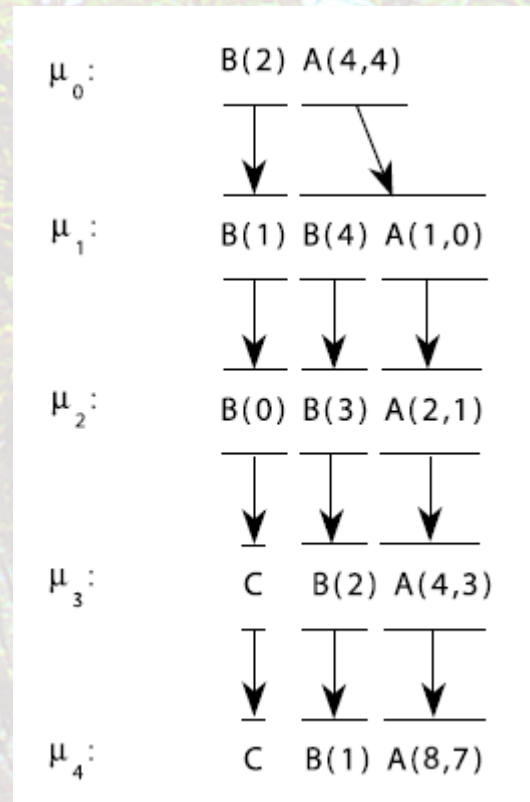
#ignore: +-F

F1F1F1

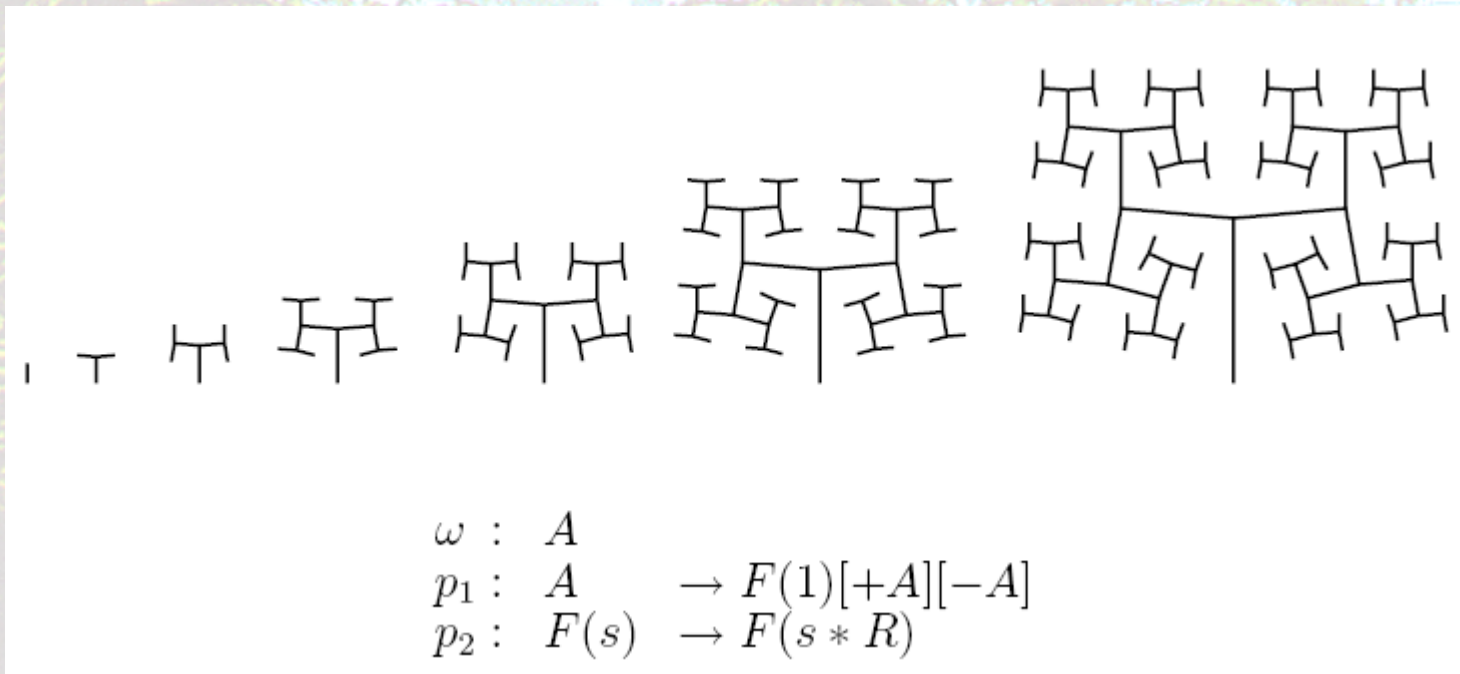
0 < 0 > 0 → 1
0 < 0 > 1 → 1[-F1F1]
0 < 1 > 0 → 1
0 < 1 > 1 → 1
1 < 0 > 0 → 0
1 < 0 > 1 → 1F1
1 < 1 > 0 → 1
1 < 1 > 1 → 0
* < + > * → -
* < - > * → +

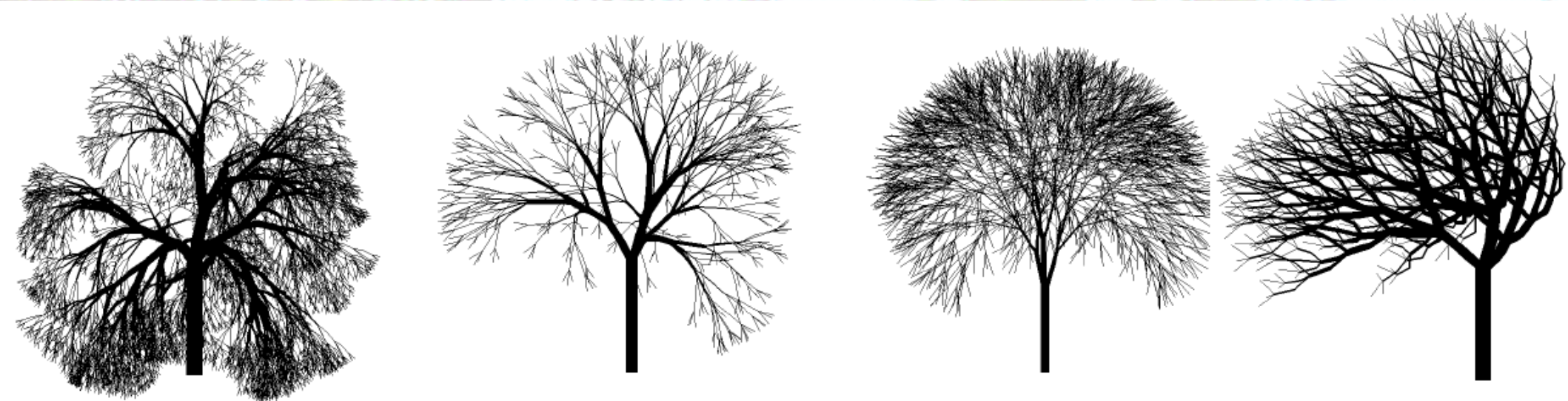
Parametrické L-systémy

- Produkčné pravidlá parametrizované a určené podmienky aplikovateľnosti

$$\begin{aligned} \omega &: B(2)A(4,4) \\ p_1 &: A(x,y) : y \leq 3 \rightarrow A(x * 2, x + y) \\ p_2 &: A(x,y) : y > 3 \rightarrow B(x)A(x/y, 0) \\ p_3 &: B(x) : x < 1 \rightarrow C \\ p_4 &: B(x) : x \geq 1 \rightarrow B(x - 1) \end{aligned}$$


Vizualizácia parametrického L-systému





```
#define d1 94.74      /* divergence angle 1 */
#define d2 132.63    /* divergence angle 2 */
#define a 18.95       /* branching angle */
#define lr 1.109     /* elongation rate */
#define vr 1.732     /* width increase rate */
```

```
ω : !(1)F(200)/(45)A
p1 : A : * → !(vr)F(50)[&(a)F(50)A]/(d1)
      [&(a)F(50)A]/(d2)[&(a)F(50)A]
p2 : F(1) : * → F(1*lr)
p3 : !(w) : * → !(w*vr)
```

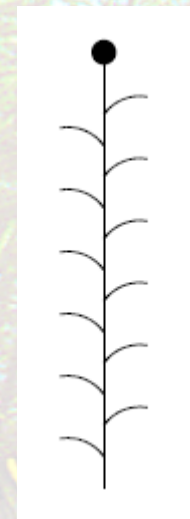


Parciálne L-systémy

- Niektoré symboly reprezentujú celé štruktúry
 - stonka “I”, kvet “K”, list “L”, púčik “A”

$$\begin{aligned}\omega &: a \\ p_1 &: a \rightarrow I[L]a \\ p_2 &: a \rightarrow I[L]A \\ p_3 &: A \rightarrow K\end{aligned}$$

a	a	a
$I[L]A$	$I[L]a$	$I[L]a$
$I[L]K$	$I[L]I[L]A$	$I[L]I[L]a$
$I[L]K$	$I[L]I[L]K$	$I[L]I[L]I[L]A$
	$I[L]I[L]K$	$I[L]I[L]I[L]K$
		$I[L]I[L]I[L]K$



Riadiace mechanizmy

- Stochastické
- Zmena tabuľky produkčných pravidiel
- Spomaľovací mechanizmus
- Spomaľovanie parametrizáciou
- Zasielanie signálov parametrizáciou

$$\begin{array}{lcl} \omega & : & a \\ p_1 & : & a \xrightarrow{\pi_1} I[L]a \\ p_2 & : & a \xrightarrow{\pi_2} I[L]A \\ p_3 & : & A \xrightarrow{1} K \end{array}$$

Table 1

$$\begin{array}{lcl} \omega & : & a \\ p_1 & : & a \rightarrow I[L]a \end{array}$$

Table 2

$$\begin{array}{lcl} p_1 & : & a \rightarrow I[L]A \\ p_2 & : & A \rightarrow K \end{array}$$

$$\begin{array}{lcl} \omega & : & a_0 \\ p_i & : & a_i \rightarrow I[L]a_{i+1} \quad 0 \leq i \leq n-1 \\ p_n & : & a_n \rightarrow I[L]A \\ p_{n+1} & : & A \rightarrow K \end{array}$$

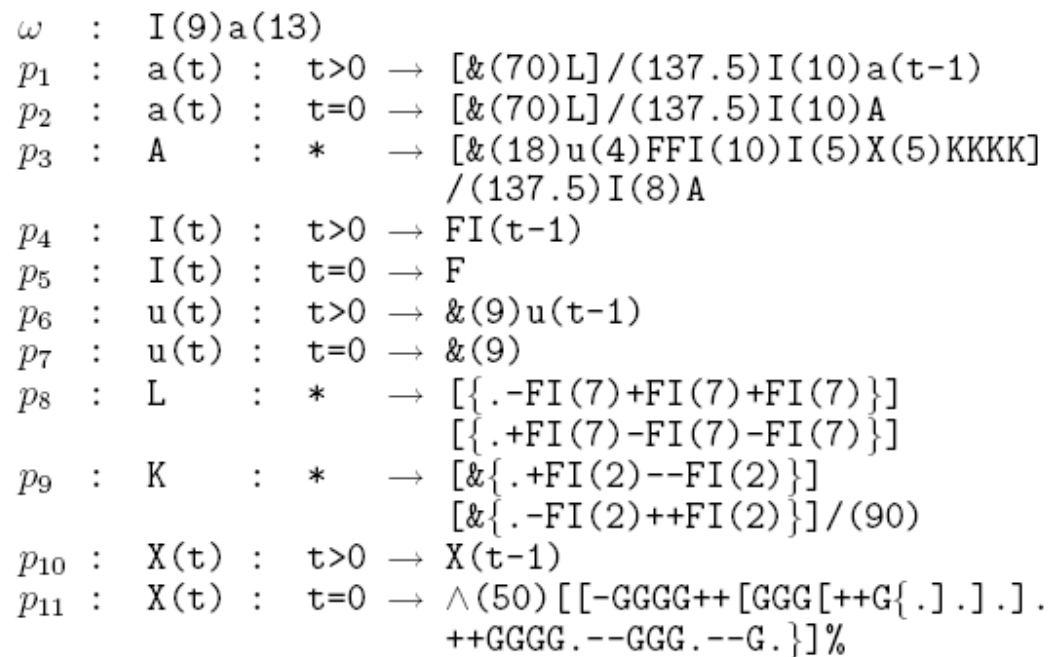
$$\begin{array}{lcl} \omega & : & a(0) \\ p_1 & : & a(c) : c < C \rightarrow I[L]a(c + \Delta c) \\ p_2 & : & a(c) : c \geq C \rightarrow I[L]A \\ p_3 & : & A : * \rightarrow K \end{array}$$

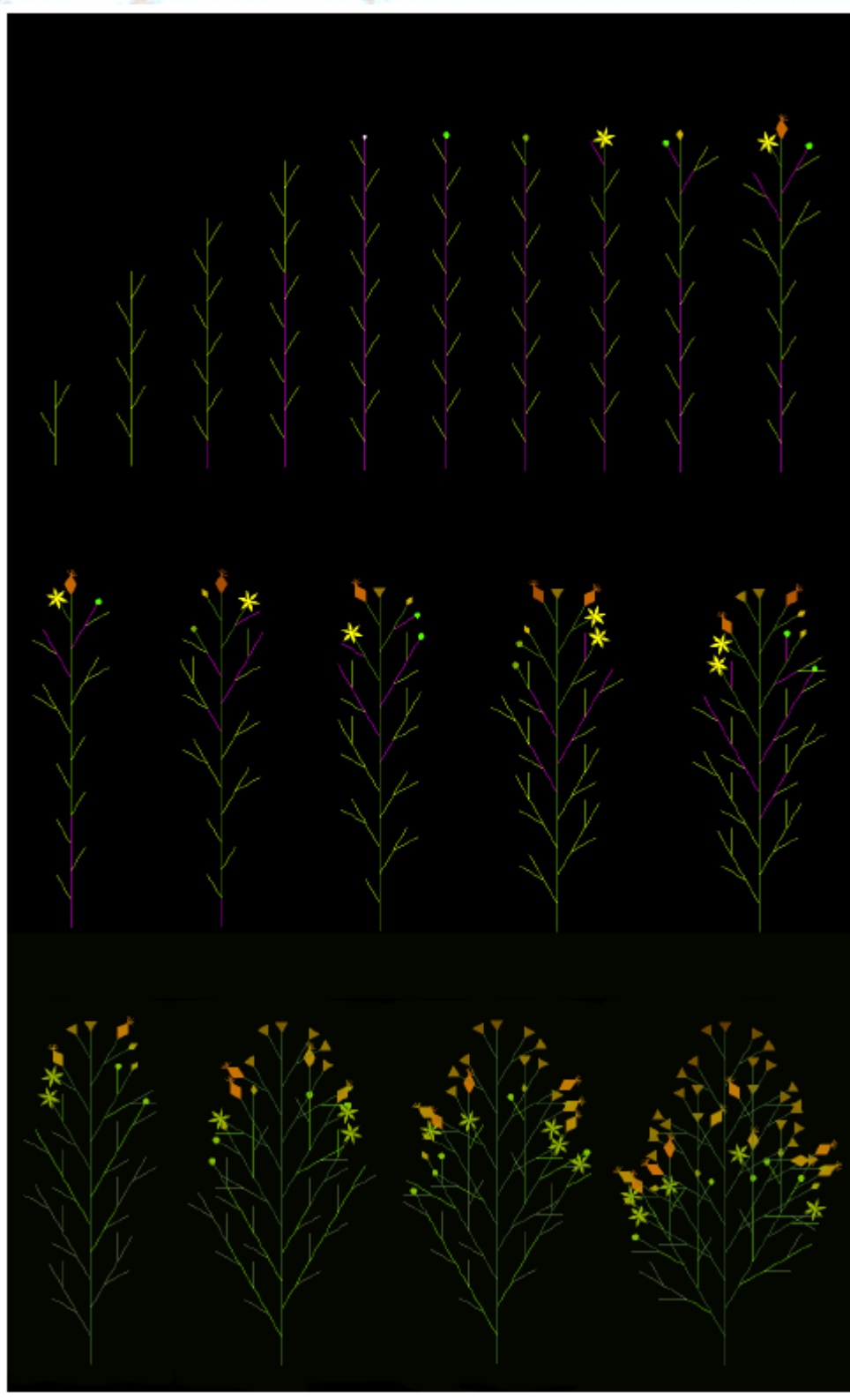
Zasielanie signálov parametrizáciou

- Umožňuje modelovať zložitejšie javy
 - Rast rastliny
 - Kvitnutie rastlín
- Príklad $d=4$, $m=2$, $u=1$

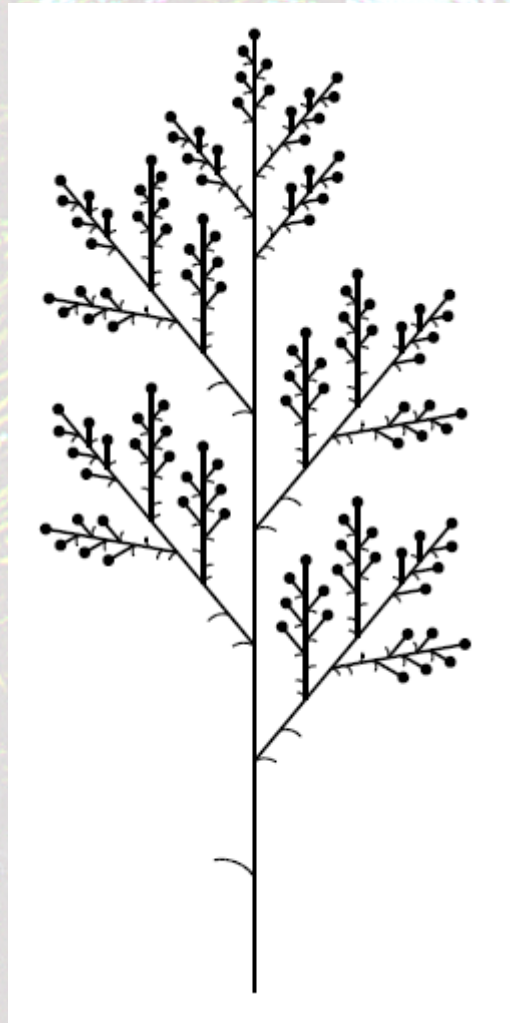
$$\begin{array}{ll}
 \omega : & D(1)a(1) \\
 p_1 : & a(i) : i < m \rightarrow a(i+1) \\
 p_2 : & a(i) : i = m \rightarrow I[L]a(1) \\
 p_3 : & D(i) : i < d \rightarrow D(i+1) \\
 p_4 : & D(i) : i = d \rightarrow S(1) \\
 p_5 : & S(i) : i < u \rightarrow S(i+1) \\
 p_6 : & S(i) : i = u \rightarrow \varepsilon \\
 p_7 : & S(i) < I : i = u \rightarrow IS(1) \\
 p_8 : & S(i) < a(j) : * \rightarrow I[L]A \\
 p_9 : & A : * \rightarrow K
 \end{array}$$

$$\begin{array}{l}
 D(1)a(1) \\
 D(2)a(2) \\
 D(3)I[L]a(1) \\
 D(4)I[L]a(2) \\
 S(1)I[L]I[L]a(1) \\
 IS(1)[L]I[L]a(2) \\
 I[L]IS(1)[L]I[L]a(1) \\
 I[L]I[L]IS(1)[L]a(2) \\
 I[L]I[L]I[L]A \\
 I[L]I[L]I[L]K
 \end{array}$$











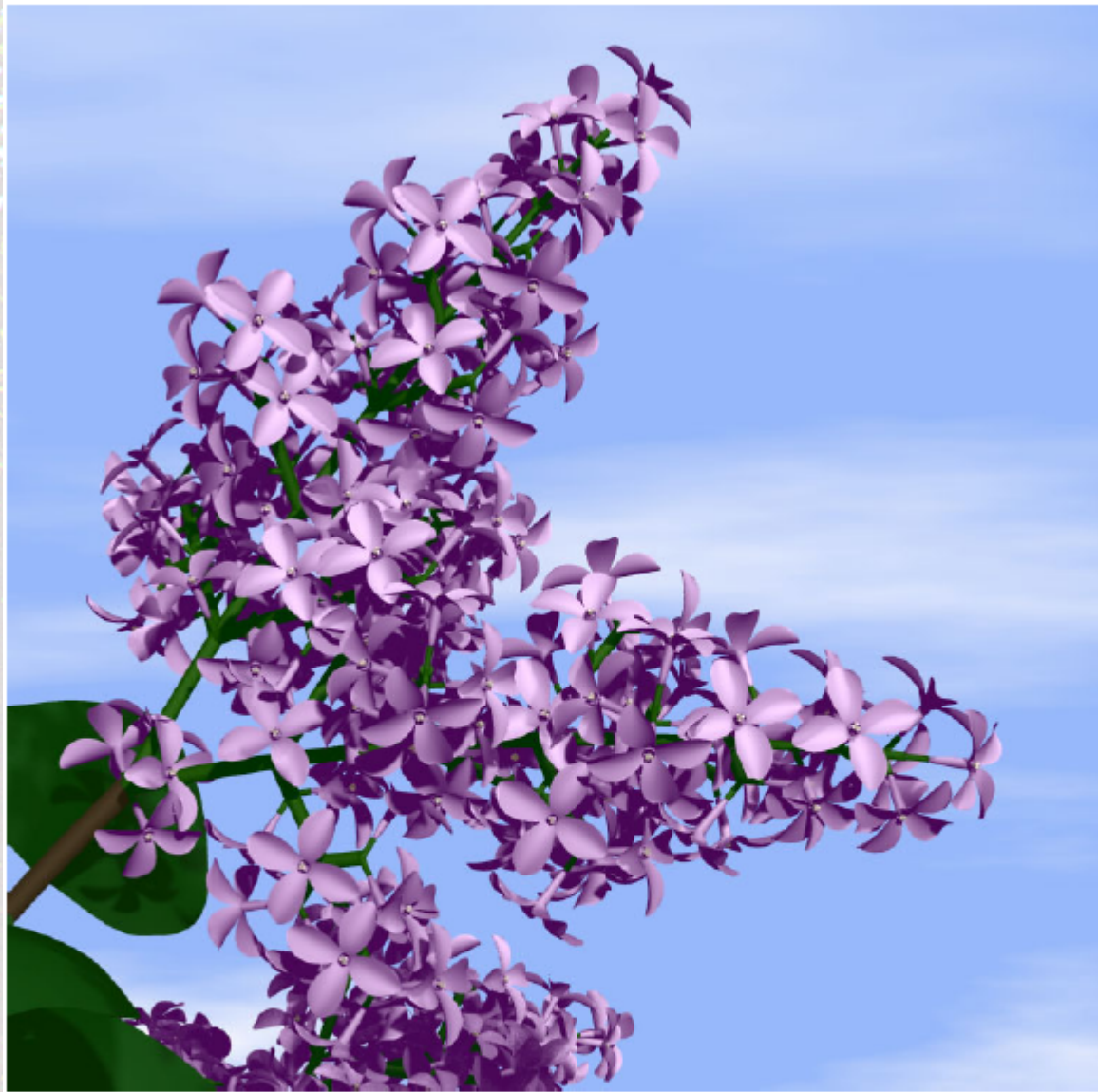
```
#define Ta 7          /* developmental switch time */
#define TL 9          /* leaf growth limit */
#define TK 5          /* flower growth limit */
#include L(0),L(1),...,L(TL) /* leaf shapes */
#include K(0),K(1),...,K(TK) /* flower shapes */
```

```
ω : a(1)
p1 : a(t) : t < Ta → F(1)[&(30)~L(0)]/(137.5)a(t+1)
p2 : a(t) : t = Ta → F(20)A
p3 : A : * → ~K(0)
p4 : L(t) : t < TL → L(t+1)
p5 : K(t) : t < TK → K(t+1)
p6 : F(l) : l < 2 → F(l+0.2)
```



```
#include L(0),L(1),... /* leaf shapes */
#include K(0),K(1),... /* flower shapes */

 $\omega$  : A(7)
 $p_1$  : A(t) : t=7  $\rightarrow$  FI(20) [&(60)~L(0)]/(90) [&(45)A(0)]/(90)
                        [&(60)~L(0)]/(90) [&(45)A(4)]FI(10)~K(0)
 $p_2$  : A(t) : t<7  $\rightarrow$  A(t+1)
 $p_3$  : I(t) : t>0  $\rightarrow$  FFI(t-1)
 $p_4$  : L(t) : *  $\rightarrow$  L(t+1)
 $p_5$  : K(t) : *  $\rightarrow$  K(t+1)
```



$n=10$, $\delta = 60^\circ$

#include K /* flower shape specification */

ω : A~K

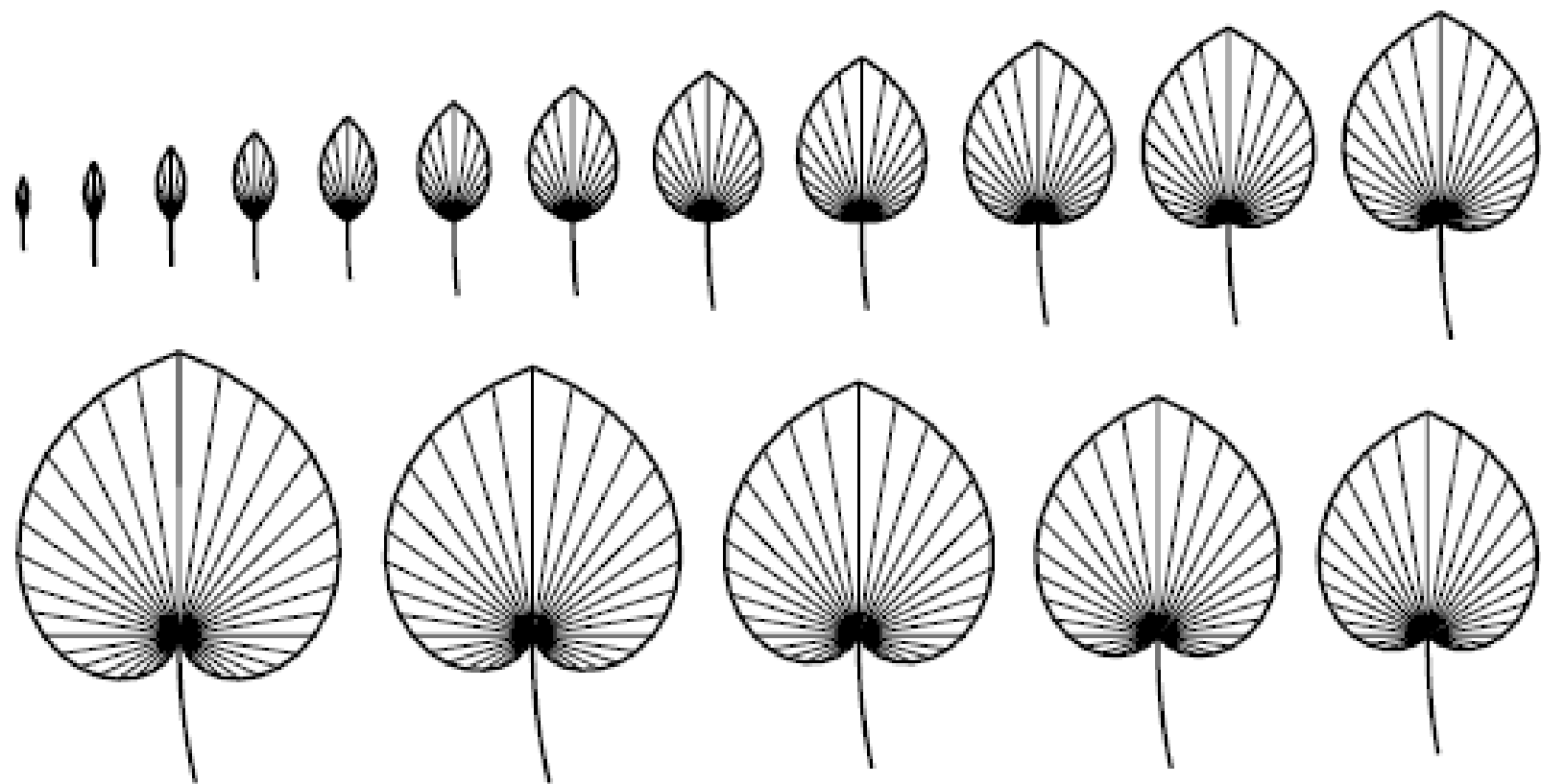
p_1 : A : * $\rightarrow [-/\sim K] [+/\sim K] I(0)/(90)A$

p_2 : I(t) : ! (t=2) $\rightarrow FI(t+1)$

p_3 : I(t) : t=2 $\rightarrow I(t+1)[-FFA][+FFA]$

Opis plošných modelov

- Opis listov, kvetov rozšíreným L-systémom
- Nové symboly na definíciu polygónov
 - začiatok nového polygónu “{“
 - uloženie pozície ako bodu polygónu “.”
 - koniec polygónu a jeho vyplnenie farbou “}”



ω : [A] [B]
 p_1 : $A \rightarrow [+A\{.\}.C.]$
 p_2 : $B \rightarrow [-B\{.\}.C.]$
 p_3 : $C \rightarrow GC$



The background of the slide is a lush, colorful floral arrangement. It features several clusters of small, five-petaled purple flowers, likely from a lilac or similar shrub, which are the most prominent elements. Interspersed among these are some pink flowers with yellow centers and two large, bright red flowers with yellow centers in the lower right corner. The flowers are set against a clear, bright blue sky. The overall composition is dense and cheerful, typical of a decorative presentation slide.

Otázky?

Použitá literatura

- **The Algorithmic Beauty of Plants [2004]**
 - Przemyslaw Prusinkiewicz
 - Aristid Lindenmayer